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Saskatchewan Curriculum

Workplace and Apprenticeship Mathematics

30

Ministry of
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Introduction

Workplace and Apprenticeship Mathematics 30 is to be allocated 100 hours. Students should receive the full amount of time allocated to their mathematical learning, and the learning should be focused upon them attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade.

The outcomes in the Workplace and Apprenticeship Mathematics 30 course are based upon students' prior learning and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These learning experiences prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

The outcomes in this curriculum define content that is considered a high priority in fields of study and areas of work for which the Workplace and Apprenticeship Mathematics pathway is appropriate. The outcomes represent the ways of thinking or behaving like a mathematics discipline area expert in those fields of study or areas of work. The mathematical knowledge and skills acquired through this course will be useful to students in many applications throughout their lives in both work and non-work settings.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things that a student needs to understand and/or be able to do to achieve the learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things students should understand or be able to do if they have attained an outcome.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Workplace and Apprenticeship Mathematics 30 outcomes are based upon the renewed Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10-12 Mathematics* (2008).

Within the outcomes and indicators in this curriculum, the terms "including" and "such as" as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, consider the outcome, "Extend and apply understanding of measures of central tendency to solve problems including mean, median, mode, weighted mean, and trimmed mean." Students are expected to determine, explain, and analyze the measures of central tendency. Other topics such as normal distribution and standard deviation, also could be explored. However, they are not an expectation of the outcome and should not be included in assessments.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, consider the indicator, "Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage, and insurance." Students need to justify a decision related to buying, leasing, or leasing to buy. However, the list of factors are only suggestions of the type of topics that students could use. Students also may include other factors that are not suggested in this indicator.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, consider the indicator, "Compare and describe, using examples, the limitations of measuring instruments used in a specific trade (e.g., tape measure versus Vernier caliper)." In this case, the listed comparison of two specific types of measurement tools is not a mandatory topic. Students who have attained this outcome and understand the limitations of measuring instruments in specific trades should be able to provide a variety of comparisons and descriptions.

Also included in this curriculum is information pertaining to how the Workplace and Apprenticeship Mathematics 30 course connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

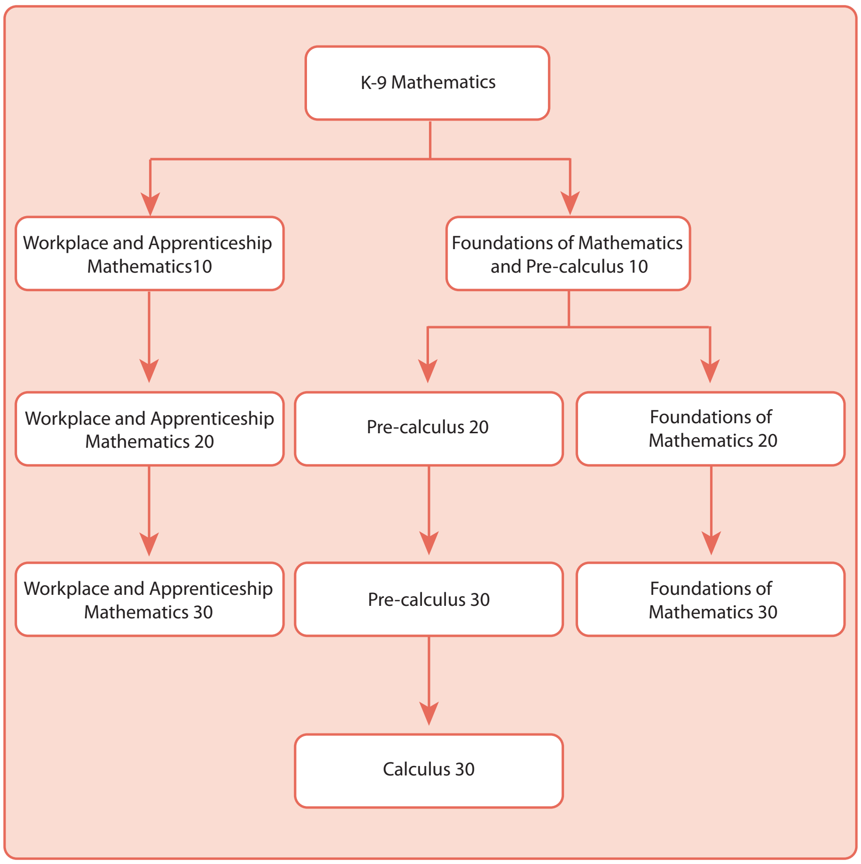
In addition, teachers will find discussions about the critical characteristics of mathematics education, inquiry in mathematics, and assessment and evaluation of student learning in mathematics.

Grades 10-12 Mathematics Framework

Saskatchewan's grades 10 to 12 mathematics curricula are based upon the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10 - 12 Mathematics* (2008). This framework was developed in response to data collected from WNCP post-secondary institutions and business and industry sectors regarding the mathematics needed by students for different disciplines, areas of study, and work areas. From these data emerged groupings of areas which required the same types of mathematics. Each grouping also required distinct mathematics, so that even if the same topic was needed in more than one of the groupings, it needed to be addressed in different ways.

The result was the creation of a set of pathways consisting of a single grade 10, 11, and 12 course for each of these groupings that were named Workplace and Apprenticeship Mathematics, Pre-calculus, and Foundations of Mathematics. During the defining of the content for these pathways and courses, it became evident that the content for Grade 10 Foundations of Mathematics and Grade 10 Pre-calculus was very similar. The result is the merging of the two Grade 10 courses (Foundations of Mathematics and Pre-calculus) into a single course entitled Foundations of Mathematics and Pre-calculus 10. The chart below visually illustrates the courses in each pathway and their relationship to each other.

10-12 Mathematics Pathway Framework



No arrows connect courses in different pathways because the content differs among the pathways. Therefore, students wishing to change pathways need to take the prerequisite courses for the pathway. For example, if students are in or have taken Pre-calculus 20, they cannot move directly into either Foundations of Mathematics 30 or Workplace and Apprenticeship Mathematics 30. In addition, if students have not taken Workplace and Apprenticeship Mathematics 10, they must do so before entering into Workplace and Apprenticeship Mathematics 20.

Each course in each pathway is to be taught and learned to the same level of rigour. No pathway or course is considered "easy math"; rather, all pathways and courses present "different maths" for different purposes.

Students may take courses from more than one pathway for credit. The current credit requirements for graduation from grade 12 are one 10 level credit and one 20 level credit in mathematics.

The Ministry of Education recommends that grade 10 students take both grade 10 courses to expose them to the mathematics in each pathway. This also will make transitions easier for those who wish to change pathways partway through their high school years.

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well, regardless of their choices after leaving school. Through its components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (2011) on the Ministry of Education website. For additional information related to the components and initiatives of Core Curriculum, please refer to the Ministry website for various policy and foundation documents.

Broad Areas of Learning

Three Broad Areas of Learning reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve understandings, skills, and attitudes related to these Broad Areas of Learning.

Lifelong Learners

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that supports their learning of new mathematical concepts and applications that they may encounter within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident

Related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Positive Lifestyle*

about their mathematical abilities, having developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

For mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 mathematics). Students must discover the mathematical knowledge outlined in the curriculum as opposed to the teacher simply covering it.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

(NCTM, 2000, p. 20)

Sense of Self, Community, and Place

To learn mathematics with deep understanding, students need to interact not only with the mathematical content, but also with each other. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue and reflection will be exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics provides many opportunities for students to enter into communities beyond the classroom through engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together.

Related to the following Goals of Education:

- *Understanding and Relating to Others*
- *Self-Concept Development*
- *Spiritual Development*

Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students, such as trends in climate change, homelessness, health issues (e.g., hearing loss, carpal tunnel syndrome, diabetes), and discrimination engage the students in interacting and contributing

Related to the following Goals of Education:

- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

positively to their classroom, school, community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for, and understanding of, differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding their roles within, and contributions to, the various communities in which they are members.

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes that are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

K-12 Goals for Developing Thinking:

- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

Related to CEL of Critical and Creative Thinking.

Within their study of mathematics, students must be engaged in personal construction and understanding of mathematical knowledge. This occurs most effectively through student engagement in inquiry and problem solving when they are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which they consider questions such as "What would happen if ...", "Could we find ...", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool that can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

K-12 Goals for Developing Identity and Interdependence:

- *Understanding, valuing, and caring for oneself (intellectually, emotionally, physically, spiritually)*
- *Understanding, valuing, and caring for others*
- *Understanding and valuing social, economic, and environmental interdependence and sustainability.*

Related to CELs of Personal and Social Development and Technological Literacy.

Given an appropriate learning environment in mathematics, students can develop both self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It also can help students take an active role in defining and maintaining the classroom environment and accepting responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning supports students in behaving respectfully towards themselves and others.

Developing Literacies

Through their mathematical learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be engaged in exploring a variety of representations for mathematical concepts and expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language are to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning.

Encouraging students to use a variety of forms of representation (concrete manipulatives; physical movement; oral, written, visual, and other symbolic forms) when exploring mathematical ideas, solving problems, and communicating understandings is important. All too often, symbolic representation is assumed to be the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper their understanding becomes.

K-12 Goals for Developing Literacies:

- *Constructing knowledge related to various literacies*
- *Exploring and interpreting the world through various literacies*
- *Expressing understanding and communicating meaning using various literacies.*

Related to CELs of Communication, Numeracy, Technological Literacy, and Independent Learning.

Developing Social Responsibility

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment constructed by the teacher and students supports respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, they become engaged in understanding the situation, concern, or issue, and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for contextual validity, and strengthened.

K-12 Goals for Developing Social Responsibility:

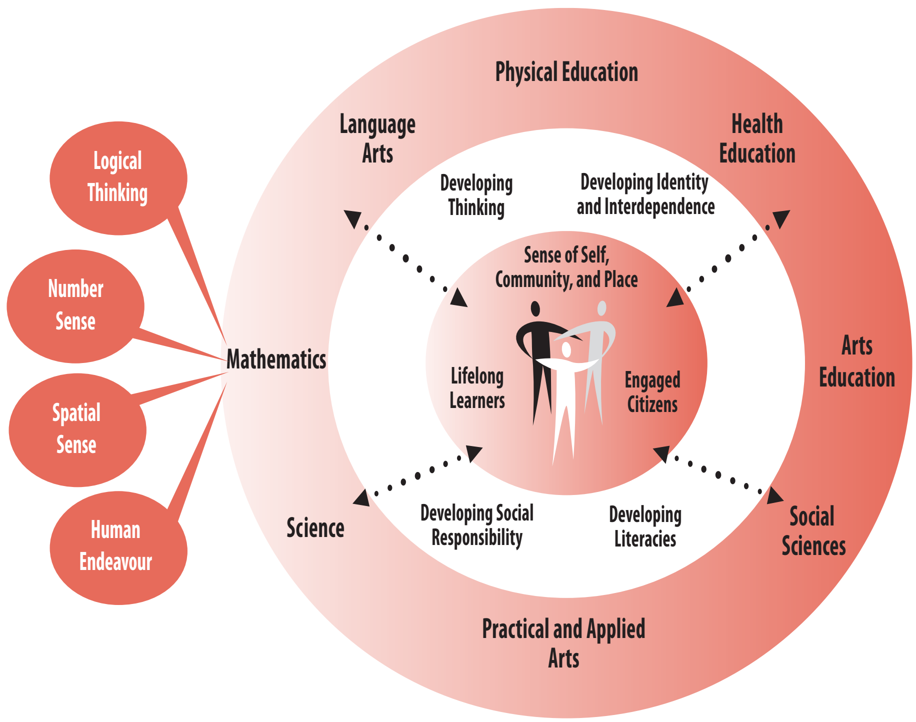
- *Using moral reasoning processes*
- *Engaging in communitarian thinking and dialogue*
- *Taking social action.*

Related to CELs of Communication, Critical and Creative Thinking, Personal and Social Development, and Independent Learning.

K-12 Aim and Goals of Mathematics

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

Defined below are four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are related directly to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes, therefore, also must promote student achievement with respect to the K-12 goals.



Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observing

- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modelling and representing (including concrete, oral, physical, pictorial, and other symbolic representations)
- conjecturing and asking "what if" (mathematical play).

To develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers, and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing numbers
- relating different operations to each other
- modelling and representing numbers and operations (including concrete, oral, physical, pictorial, and other symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, for students to become flexible and confident in their calculation abilities, and to be able to transfer those abilities to more abstract contexts, students first must develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

The ability to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation and testing of conjectures based upon discovered patterns drive the students' development of spatial sense, with formulas and definitions resulting from their mathematical learnings.

Meaning does not reside in tools; it is constructed by students as they use tools.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Hunman, 1997, p. 10)

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematical and personal)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet its particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments, and those developments are determined by the contexts and needs of the time, place, and people.

All students benefit from mathematics learning that values and respects different ways of knowing mathematics and its relationship to the world. The mathematics content found within this curriculum often is viewed in schools and schooling through a Western or European lens, but there are many different lenses, such as those of many First Nations and Métis peoples, through which mathematics can be viewed and understood. The more exposure that all students have to differing ways of understanding and knowing mathematics, the stronger they will become in their number sense, spatial sense, and logical thinking.

The content found within the grade level outcomes for the K-12 mathematics program and its applications first and foremost is the vehicle through which students can achieve the four K-12 goals of mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula, there is very little to cover but an awful lot to uncover [discover]". This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience reveal complex, interrelated set of characteristics that teachers need to be aware of to provide an effective mathematics program.

Assumptions in this Curriculum

The question in mathematics often arises as to whether students should work with fractions, decimals, or both, and if working with fractions, whether mixed numbers or improper fractions be used. For the purposes of this document, we assume the following:

- If a question or problem is stated with fractions (decimals), the solution should involve fractions (decimals), unless otherwise stated.
- Final fraction solutions can be stated in mixed numbers or improper fractions as long as this is consistent with the original stating of the question or problem.
- The word "or" indicates that students should be able to work with the list of strategies, representations, or approaches given in the list, but they should not be expected to apply more than one of such strategies, representations, or approaches to any given situation or question. For example, in the indicator, "Research and present, orally, in writing, or using multimedia, applications of transformations using examples and illustrations in construction, industrial, commercial, domestic, and artistic contexts."

When engaging in activities related to graphing, the word "sketch" indicates that the graph can be produced without the use of specific tools or an emphasis on precision. The word "draw" indicates that specific tools (such as graphing software or graph paper) should be used to produce a graph of greater accuracy.

Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from "covering" the outcomes to supporting students in "discovering" mathematical concepts. These facets include:

- the organization of the outcomes
- the seven mathematical processes
- the difference between covering and discovering mathematics
- the development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiquing statements
- the concrete to abstract continuum
- modelling and making connections
- the role of homework
- the importance of ongoing feedback and reflection.

Organization of Outcomes

The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are a succinct set of high-level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses.

For ease of reference, the outcomes in this document are numbered using the following system: WA30.#, where WA refers to Workplace and Apprenticeship Math, 30 indicates the course level, and # is the number of the outcome in the list of outcomes. WA30.1 need not be taught before WA30.11.

Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics, rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Workplace and Apprenticeship Mathematics 30 are at the discretion of the teacher.

Mathematical Processes

This Workplace and Apprenticeship Mathematics 30 curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on communicating, making connections, mental mathematics and estimating, problem solving, reasoning, visualizing, and using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in mathematics should be addressed through the appropriate mathematical processes, as indicated by the bracketed letters following each outcome. During planning, teachers should carefully consider those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, prior knowledge, the formal language and symbols of mathematics, and new learning.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology once they have had sufficient experience to develop an understanding of that terminology.

Concrete, pictorial, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.

(Caine & Caine, 1991, p. 5)

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you ...?", "Can you ...?", or "What if ...?", the problem solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

Mathematical problem-solving often involves moving backwards and forwards between numerical/ algebraic representations and pictorial representations of the problem.

(Haylock & Cockburn, 2003, p. 203)

For an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students are given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for and engage in finding a variety of strategies for solving problems empowers them to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. Meaningful inquiry challenges students to think and develop a sense of wonder about mathematics.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.

(NCTM, 2000, p. 191)

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.

(Armstrong, 1993, p. 10)

Technology should not be used as a replacement for basic understandings and intuition.

(NCTM, 2000, p. 25)

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Technology [T]

Technology tools contribute to student achievement of a wider range of mathematics outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. Students should understand and appreciate the appropriate use of technology in a mathematics classroom. Students also should learn to distinguish when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding but rather enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social convention or custom of mathematics.

This content includes what the symbol for an operation looks like, mathematical terminology, and conventions regarding the recording of symbols and quantities.

Content that can and should be discovered by students is that which can be constructed by students based on prior mathematical knowledge. This content includes things such as strategies, processes, and rules as well as the students' current and intuitive understanding of quantity, patterns, and shapes. Any learning in mathematics that is a consequence of the logical structure of mathematics can and should be constructed by students.

For example, when learning in relation to outcome WA30.4,

> Extend and apply understanding of the properties of triangles, quadrilaterals, and regular polygons to solve problems.

students can explore, describe, and generalize a variety of properties pertaining to triangles, quadrilaterals, and regular polygons. Recognizing patterns and relationships in polygons allows students to discover that diagonals in a rhombus are perpendicular, rather than the teacher simply telling this property to the students. In addition, they can identify and explore authentic situations involving polygons such as the appearance of hexagons when two bubbles touch or in beehives, turtle shells, and snowflakes. Engaging students in the development of these understandings expands their abilities related to number sense, spatial sense, logical thinking, and understanding mathematics as a human endeavour (the four goals of K-12 mathematics).

Development of Mathematical Terminology

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that they connect the new terminology with their developing mathematical understanding. Therefore, students first must linguistically engage with new mathematical concepts using words that are already known or that make sense.

Teachers should model appropriate conventional vocabulary.

(NCTM, 2000, p. 131)

For example, in outcome WA30.9,

> Extend and apply understanding of measures of central tendency to solve problems including:
>
> - mean
> - median
> - mode
> - weighted mean
> - trimmed mean.

the terms “weighted mean” or “trimmed mean” may be new to the students. Rather than providing them with a textbook definition, students can construct understanding by developing the meaning of “weighted mean” and “trimmed mean” through instructional strategies such as concept attainment. After the class has come to an agreement about how to define the unknown characteristic that the concept attainment activity is focusing on, then the words “weighted mean” and “trimmed mean” can be introduced as mathematical terminology. At this point, students can explore data sets to make connections between the data and their "weighted" or "trimmed" means to construct and deepen understanding of statistical applications. Finally, students then are prepared to consider published definitions and critique them.

To help students develop their working mathematical language, teachers must recognize that many students, including First Nations and Métis, may not recognize a specific term or procedure but may, in fact, have a deep understanding of the mathematical topic. Many perceived learning difficulties in mathematics are the result of students’ cultural and personal ways of knowing not being connected to formal mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. Students should be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

Teachers must recognize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understanding. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught in relation to their schema, cultural and environmental context, or real life experiences.

A first step in the actualization of mathematics from First Nations and Métis perspectives is empowering teachers to understand that mathematics is not acultural. As a result, teachers realize that the traditional Western European ways of teaching mathematics also are culturally biased. These understandings will support the teacher in developing First Nations and Métis students’ personal mathematical understanding and mathematical self-confidence and ability through a more holistic and constructivist approach to teaching. Teachers need to pay close attention to these factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

Teachers must recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding, and as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas among cultural groups cannot be assumed to be a direct link. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. In addition, they also allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. As well, ethnomathematics demonstrates the relationship between mathematics and cultural anthropology.

Individually and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to themselves personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of Elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. In doing so, the teacher can identify strengths and deficiencies in students' understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, consider the indicator,

> Critique the statement: "Predictions based on percentile ranks are always 100% accurate".

While a percentile rank can assist in interpreting how an individual piece of data fits into the framework of the whole group, it is not used as a 100% accurate prediction of future events. By asking students to critique this statement, teachers can become aware of the preconceptions students hold toward percentile rank, if any, and the application of statistics. Teachers could use height and weight measurements of star athletes to engage students in determining how a Body Mass Index (BMI) predicts whether that athlete is underweight, normal, overweight, or obese. Presenting authentic situations allows students to analyze the role of percentile rank and the value of making predictions. Students might also draw reference to percentile growth charts in order to illustrate common applications of percentile rank, how they are used to make predictions on the future growth pattern of a child, and what impact that prediction may or may not have with doctors and parents.

Critiquing statements is an effective way to assess students individually, as a small group or as a large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important for students to use representations that are meaningful to them.

(NCTM, 2000, p. 140)

It is important, in learning mathematics, that students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a more concrete starting point. Therefore, teachers must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving surface area: What is the surface area of your computer? Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives or pictures), or both.

Models and Connections

New mathematics is developed continuously by creating new models as well as combining and expanding existing models. Although the final products of mathematics most frequently are represented by symbolic models, their meaning and purpose often are found in the concrete, physical, pictorial, and oral models, and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and other symbolic models). In addition, students need to make connections between the different representations. These connections are made by having students move from one type of representation to another (i.e., how could you represent what you've done here using mathematical symbols?) or by having them compare their representations with others in the class. In making these connections, students can reflect upon the mathematical ideas and concepts that are being used in their new models.

Making connections also involves looking for patterns. For example, in outcome WA30.3:

> Solve problems that involve the sine law and cosine law, excluding the ambiguous case.

In the past, students have analyzed Pythagorean Triples and used this relationship to solve problems involving two right triangles. Students can continue to make connections to the Pythagorean Theorem by extending into developing the law of cosines. Students also may choose to explore, describe, and make generalizations about the trigonometric relationships from within obtuse and acute triangles while utilizing prior knowledge of right angle triangles.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that consolidate new learnings with prior knowledge, explore possible solutions, and apply learning to new situations. Although drill and practice serve a purpose in learning for deep understanding, the amount and timing of drill varies among different learners. In addition, when used as homework, drill and practice frequently causes frustration, misconceptions, and boredom to arise in students.

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.

(NCTM, 2000, p. 139)

Characteristics of Good Math Homework

- *It is accessible to [students] at many levels.*
- *It is interesting both to [students] and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to, and types of, math homework offered to [students] throughout the year.*

(Adapted from Raphel, 2000, p. 75)

As an example of the type or style of homework that can help students develop deep understanding in Workplace and Apprenticeship Mathematics 30, consider outcome WA30.7:

> Explore and critique the viability of small business options with respect to:
>
> - expenses
> - sales
> - profit or loss.

Rather than telling students about the financials of a small business, invite students (pairs or individuals) to prepare a presentation that may or may not involve technology, for a meeting with a small business owner and a financial institution. Students could visit the owner of a small business, discuss options with parents or others, or search the Internet. The presentations could be made to a panel that grants small business loans in relation to start-up capital, expansion, or rejuvenation of the small business. After each presentation, the class can discuss what criteria contributed to the feasibility of the small business within the given community context. Prior to the assignment, a rubric can be developed to guide students in the expectations of the assignment and encourage self-assessment. Such an assignment encourages oral discussion, provides a venue to develop research, organizational, and presentation skills; and develops the mathematical language necessary for understanding aspects of being an entrepreneur.

Ongoing Feedback and Reflection

Feedback can take many different forms. Instead of saying, "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about ...?"

(Stiff, 2001, p. 70)

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires both the teacher and students to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understanding. Feedback from students to teachers provides much needed information to planning for further and future learning.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what students do and do not know. Through such reflections, not only can a teacher make better informed instructional decisions, but also a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, students must learn by constructing knowledge, with very few ideas relayed directly by the teacher. As an example, the teacher will have to show and name function notations for the students; however, first, the students could explore those ideas important for working with function notation.

Teachers should analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to provide opportunities for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and the planning of effective strategies to promote students' deeper understanding of mathematical ideas.

A mathematics learning environment should include an effective interplay of:

- reflecting
- exploring patterns and relationships
- sharing ideas and problems
- considering different perspectives
- decision making
- generalizing
- verifying and proving
- modelling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. Conversely, when they learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.

(NCTM, 2000, p. 72)

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences .... A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.

(Haylock & Cockburn, 2003, p. 18)

What might you hear or see in a Workplace and Apprenticeship 30 classroom that would indicate to you that students are developing a deep understanding?

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.

(Mills & Donnelly, 2001, p. xviii)

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

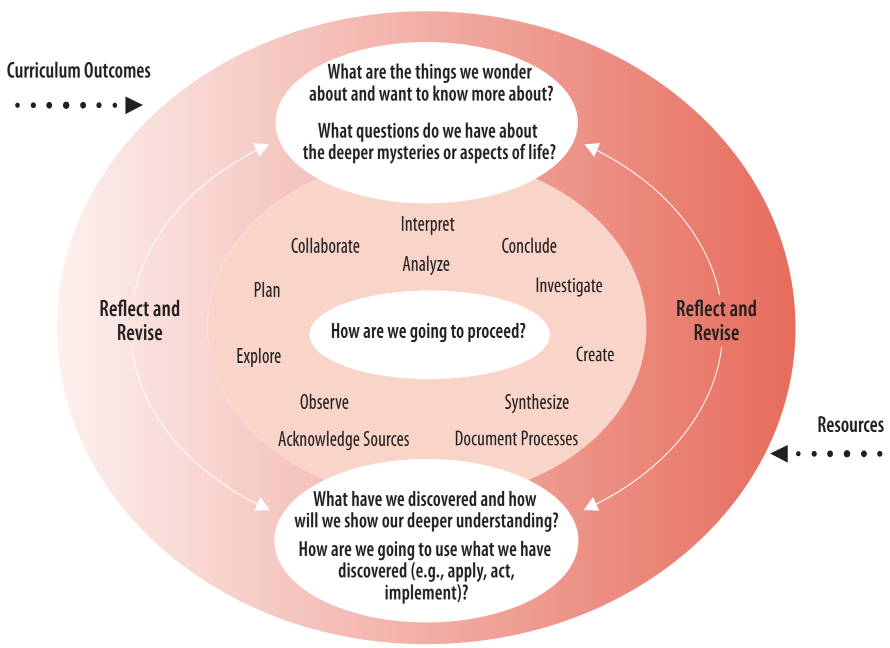
Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are involved and engaged directly in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process but rather a cyclical one, with parts of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic demonstrates the cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as they become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, develop conclusions, document and reflect on learning, and generate new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional similar problems, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation already has been determined, it no longer is problem solving. Students must understand this difference too.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points. However, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. Develop questions evoked by students' interests have the potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry, and give students direction for discovering deep understanding about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When or why might you want to use a transformation of a 2-D shape or 3-D object?"
- "How do you know when you have an answer?"
- "Will this strategy work for all situations?"
- "How does your representation compare to that of your partner?"

Effective questions:

- *cause genuine and relevant inquiry into the important ideas and core content*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answer*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons*
- *spark meaningful connections with prior learning and personal experiences*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning. Questioning also should be used to encourage students to reflect on the inquiry process and on the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- help students make sense of the mathematics.
- are open-ended, whether in answer or approach, as there may be multiple answers or multiple approaches.
- empower students to unravel their misconceptions.
- not only require the application of facts and procedures but encourage students to make connections and generalizations.
- are accessible to all students and offer an entry point for all students.
- lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.

(Schuster & Canavan Anderson, 2005, p. 3)

Reflection and Documentation of Inquiry

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.

(Schuster & Canavan Anderson, 2005, p. 1)

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics can take the form of reflective journals, notes, models, works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understanding.

Students must engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen their understanding.

Outcomes and Indicators

Goals: Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.1 Analyze puzzles and games that involve logical reasoning using problem-solving strategies. **[C, CN, PS, R]**	**Note:** This outcome is intended to be integrated throughout the course by using logical puzzles and games such as Chess, Sudoku, Mastermind, Nim, Reversi. a. Determine, explain, and verify strategies to solve a puzzle or to win a game such as: • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • formulate and simplify a problem that is similar to the original problem • work backwards • develop alternative approaches. b. Observe and analyze errors in solutions to puzzles or in strategies for winning games, and explain the reasoning. c. Create a variation on a puzzle or a game, and describe a strategy for solving the altered puzzle or winning the game.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.2 Demonstrate concretely, pictorially, and symbolically an understanding of limitations of measuring instruments including: • **precision** • **accuracy** • **uncertainty** • **tolerance.** **[C, PS, R, T, V]**	a. Explain, using concrete models and pictorial representations, the difference between precision and accuracy. b. Analyze given contexts to generalize and explain why: • a certain degree of precision is required • a certain degree of accuracy is required. c. Compare the degree of accuracy of two or more given instruments used to measure the same attribute. d. Relate the degree (margin) of accuracy to the uncertainty of a given measure.

Outcomes	Indicators
WA30.2 continued	e. Analyze and justify the degree of precision and accuracy required in contextual problems. f. Analyze given contexts to calculate maximum and minimum values, using a given degree (range) of tolerance. g. Compare and describe, using examples, the limitations of measuring instruments used in a specific trade or industry, (e.g., tape measure versus Vernier caliper). h. Create and solve situational questions that involve precision, accuracy, or tolerance, and explain the reasoning and the strategy used to arrive at the solution.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.3 Solve problems that involve the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]	a. Identify and describe the use of the sine law and cosine law in construction, industrial, commercial, and artistic applications. b. Solve situational questions that involve the sine law or cosine law.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.4 Extend and apply understanding of the properties of triangles, quadrilaterals, and regular polygons to solve problems. [C, CN, PS, V]	a. Analyze, generalize, and explain properties of polygons using illustrations, including: • triangles (isosceles, equilateral, scalene, and right triangles) • quadrilaterals in terms of angle measures, side lengths, diagonal lengths, and angles formed by the intersection of diagonals • regular polygons. b. Explain, using examples, why a given property does or does not apply to certain polygons (e.g., the diagonals of a square are perpendicular, but the diagonals of a rectangle are not even though squares are rectangles). c. Identify and explain applications of the properties of polygons in construction, industry, commerce, domestic, and artistic contexts. d. Create and solve situational questions that involve the application of the properties of polygons.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.5 Extend and apply understanding of transformations on 2-D shapes and 3-D objects including: • **translations** • **rotations** • **reflections** • **dilations.** **[C, CN, R, T, V]**	a. Analyze original 2-D shapes and 3-D objects and their images to identify and justify the single transformation that was performed. b. Draw the image of 2-D shapes given: • a single transformation including a translation, rotation, reflection, and justify why it is a translation, rotation, or reflection. • a combination of successive transformations and explain the reasoning. c. Create designs using translations, rotations, and reflections in all four quadrants of a coordinate grid. d. Analyze and describe designs that involve translations, rotations, and reflections in all four quadrants of a coordinate grid, and explain the reasoning. e. Research and present, orally, in writing, or using multimedia, applications of transformations using examples and illustrations in construction, industrial, commercial, domestic, and artistic contexts. f. Analyze and generalize the relationship between reflections and lines or planes of symmetry. g. Explain how and why the concept of similarity can be used to determine if an image is a dilation of a given shape, and provide examples. h. Determine whether or not given images are dilations of given shapes and explain the reasoning. i. Draw, with or without technology, a dilation image for a given 2-D shape and 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional. j. Solve contextual problems that involve transformations and explain the reasoning.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.6 Demonstrate understanding of options for acquiring a vehicle including: • **purchasing without credit** • **purchasing with credit** • **leasing** • **leasing to purchase.** **[C, CN, PS, R, T]**	a. Research and present various options for purchasing or leasing a vehicle (oral, written, multimedia, etc.). b. Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage, and insurance. c. Solve, with or without technology, situational questions that involve the purchase, lease, or lease to purchase of a vehicle.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.7 Explore and critique the viability of small business options with respect to: • **expenses** • **sales** • **profit or loss.** **[C, CN, R]**	a. Analyze small businesses such as a hot dog stand to identify and describe expenses, and explain factors, such as seasonal variations and hours of operation that might impact their profitability. b. Research and describe feasible small business options for a given community. c. Analyze a small business to generate options that might improve its profitability, and report to an audience. d. Determine the break-even point for small businesses and explain the reasoning.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes

WA30.8 Extend and apply understanding of linear relations including:

- **patterns and trends**
- **graphs**
- **tables of values**
- **equations**
- **interpolation and extrapolation**
- **problem solving.**

[CN, PS, R, T, V]

Indicators

a. Analyze graphs, tables of values, number patterns, and/or equations to generalize characteristics of linear relations.

b. Analyze relations in sets of graphs, tables of values, number patterns, and/or equations to sort according to whether the relations are linear or nonlinear.

c. Represent and explain the linear relation in given contexts, including direct or partial variations, using equations, tables of values, and/or sketches of graphs.

d. Analyze contexts and their graphs to explain why the points on the graphs should or should not be connected.

e. Create, with or without technology, a graph to represent a data set, including scatterplots.

f. Analyze graphs of data sets, including scatterplots, to generalize and describe the trends.

g. Analyze and sort a set of scatterplots according to the trends represented (linear, nonlinear, or no trend).

h. Critique statements such as "Trends allow us to predict exactly what will happen in the near future."

i. Solve situational questions that require interpolation or extrapolation of information.

j. Relate slope and rate of change to linear relations.

k. Match given contexts with their corresponding graphs and explain the reasoning.

l. Create and solve situational problems that involve the application of a formula for a linear relation.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.9 Extend and apply understanding of measures of central tendency to solve problems including: • **mean** • **median** • **mode** • **weighted mean** • **trimmed mean.** [C, CN, PS, R]	a. Explain, using examples, the advantages and disadvantages of each measure of central tendency. b. Determine the mean, median, and mode for sets of data and explain the reasoning. c. Analyze calculations of measures of central tendency to identify and correct errors if necessary. d. Critique statements such as "It is not possible to have a set of data which displays a mean, a median, and a mode of the same value." e. Identify the outlier(s) in a set of data, explain why they are outliers, and discuss their effect on the mean, median, and mode of that data set. f. Calculate the trimmed mean for sets of data and justify the removal of the outliers. g. Explain, using examples such as course marks, why some data in a set would be given a greater weighting in determining the mean. h. Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean) and explain the reasoning. i. Explain, using examples from print and other media, how and why measures of central tendency and outliers are used to provide different interpretations of data. j. Create and solve situational questions that involve measures of central tendency.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.10 Demonstrate understanding of percentiles. [C, CN, PS, R]	a. Explain, using examples, percentile ranks in a context. b. Explain how and why decisions can be made based on percentile rank. c. Compare, using examples, percent and percentile rank. d. Analyze and generalize the relationship between median and percentile. e. Solve situational questions that involve percentiles and percentile charts.

Outcomes	Indicators
WA30.11 continued	f. Critique statements such as "Predictions based on percentile ranks are always 100% accurate."

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
WA30.11 Extend and apply understanding of probability. **[C, CN, PS, R]**	a. Research and present orally, in writing, or using multimedia, applications of probability (e.g., medication, warranties, insurance, lotteries, weather prediction, 100-year flood, failure of a design, failure of a product, vehicle recalls, approximation of area). b. Calculate the probability of an event based on a data set, (e.g., determine the probability of a randomly chosen light bulb being defective). c. Express given probabilities as fractions, decimals, percentages, and using words. d. Analyze, generalize, and compare odds and probability including part-whole and part-part relationships. e. Determine the probability of an event, given the odds for or against. f. Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, results of experimental probability, and subjective judgments. g. Solve situational questions that involve probability. h. Critique statements such as, "It is not possible to express odds as fractions".

Assessment and Evaluation of Student Learning

Assembling evidence from a variety of sources is more likely to yield an accurate picture.

(NCTM, 2000, p. 24)

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement is based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- achievement of provincial curriculum outcomes
- effectiveness of teaching strategies employed
- student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information unrelated to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Workplace and Apprenticeship Mathematics 30. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment should not merely be done to students; rather it should be done for students.

(NCTM, 2000, p. 22)

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

What are examples of assessments as learning that could be used in Workplace and Apprenticeship Mathematics 30, and what would be the purpose of those assessments?

Assessment as learning involves student reflection on and monitoring of her/his progress related to curricular outcomes, and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement, and:

- provides an opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be engaged regularly in assessment as learning. The various types of assessments should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for them to set and assess personal learning goals related to the content of Workplace and Apprenticeship Mathematics 30.

Assessment should become a routine part of the ongoing classroom activity rather than an interruption.

(NCTM, 2000, p. 23)

Glossary

2-D Shapes: The face of a 3-D object. 2-D shapes do not exist without a 3-D object of which they are a part.

Accuracy: The difference between a measurement reading and the true value of that measurement. For example, a measurement of 1.23 metres for the length of a particular object may be fairly accurate, while a measurement of 1.234 is more precise but may be less accurate.

Break-even point: The point where the total revenue is sufficient just to cover the total cost, that is, the **volume** of **sales** at which a **company's net sales** just equals its **cost** or the gains equal the losses.

Direct variation: A relationship between two variables in which one is a constant multiple of the other. When y is directly proportional to x, the equation is of the form y = mx (where m is a constant). The line of the graph passes through the origin.

Leasing: Leasing is a method of paying an agreed-upon monthly payment for the use of a vehicle over a specified period of time. Unlike renting a vehicle for as little as a day or even a few hours, leasing typically starts at 24 months and does not provide for easy termination or vehicle swapping.

Generalize: The process of describing in general patterns and/or processes from specific examples and cases. Frequently, generalizing is an inductive process, but it also can involve deductive proof of the pattern or process.

Graphic organizer: A pictorial or concrete representation of knowledge, concepts, and/or ideas, and the connections among them.

Leasing to purchase: The right to buy a vehicle that has been leased at the end of the lease term for a stated price.

Odds: The probable number of times that an event is likely to occur, expressed as a ratio of the number of favourable outcomes to the number of unfavourable outcomes (probability : 1 - probability).

Partial variation: A relationship between two variables in which one variable is a constant multiple of the other plus a constant value. A partial variation is represented by the equation of the form y = mx + b. The x and y values do not vary directly with each other and the graph does not pass through the origin (for example, the cost of renting a car is the basic daily charge plus a charge per kilometre driven; shown as C = .25K + 20.00).

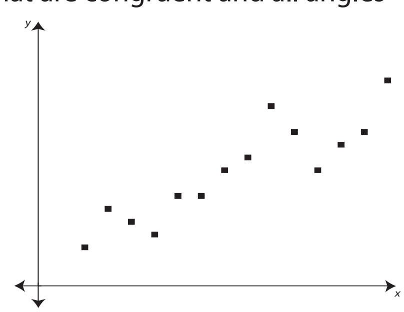
Precision: The precision of a measuring instrument is determined by the smallest unit that it can measure. Precision is said to be the same as the smallest fractional or decimal division on the scale of the measuring instrument. A precise number has many significant digits. For example, a measurement given as 12.14 +/- 0.01 cm is more precise than one given as 12 +/- 1 cm. Higher precision does not necessarily imply higher accuracy.

Referent: A personally determined concrete or physical approximation of a quantity or unit of measurement. For example, some people use the width of their thumb as a referent for one inch.

Regular polygons: A closed plane figure for which all sides are line segments that are congruent and all angles are congruent.

Scatterplot: A graph in which the data is displayed as a collection of points, each having the value of one variable determining the position on the horizontal axis and the value of the other variable determining the position on the vertical axis as (x,y) points.

Trimmed mean: A method of averaging that removes a certain percentage of the lowest and highest values before computing the mean of the remaining scores to avoid the effects of outliers.



Scatterplot

Uncertainty: No measurement is exact because when a quantity is measured, the outcome depends on the measuring system, the measurement procedure, the skill of the operator, the environment, and other effects. This uncertainty or variation in a measurement often is described as the error or a mathematical way to show the uncertainty in the measurement. Uncertainty is the potential difference between the result of the measurement and the true value of what is being measured.

Vernier caliper: A device for linear measurement that is used to measure objects that may be difficult because of the small length, such as the interior and exterior of pipes. A micrometer is a more precise vernier caliper that measures a lesser range of distance.

Weighted mean: An average in which the elements of the set are assigned a weight so that some values carry more importance (value or weight) than others.

US Customary System: The most commonly used system of measurement in the United States. Like the Imperial system, it is based upon the historical system of measurement known as the English units. The US Customary system differs from the Imperial system in some measurements of volume, mass, and length.

Tolerance: The unwanted but acceptable deviation from desired measurement. It is the greatest range of variation or error that can be allowed.

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Workplace and Apprenticeship Mathematics 30 Curriculum

1. Please indicate your role in the learning community

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☐ other ____

What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

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3. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
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