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Saskatchewan Curriculum

Pre-calculus

30

Ministry of
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Introduction

Pre-calculus 30 is to be allocated 100 hours. Students must receive the full amount of time allocated to their mathematical learning and the learning should be focused upon them attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade.

The outcomes in the Pre-calculus 30 course are based upon students' prior learning and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These learning experiences prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

The outcomes in this curriculum define content that is considered a high priority in fields of study and areas of work for which the Pre-calculus pathway is appropriate. The outcomes represent the ways of thinking or behaving like a mathematics discipline area expert in those fields of study or areas of work. The mathematical knowledge and skills acquired through this course will be useful to students in many applications throughout their lives in both work and non-work settings.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things that a student needs to understand and/or be able to do to achieve the learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things students should understand or be able to do if they have attained an outcome.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Pre-caclulus 30 outcomes are based upon the renewed Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10-12 Mathematics* (2008).

Within the outcomes and indicators in this curriculum, the terms "including" and "such as" as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, consider the indicator, "Solve situational questions that involve exponential growth or decay, including in loans, mortgages, and investments." Students, along with exploring a variety of exponential growth or decay problems, are expected to include loans, mortgages and investments.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, consider the indicator, "Develop and apply strategies such as lists or tree diagrams, to determine the total number of choices or arrangements possible in a situation." Students are not limited to using lists or tree diagrams and may choose to use different graphic organizers to represent the total number of arrangements. Students are not required to use any of these strategies in particular.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, consider the indicator, "Explain how to estimate the value of a logarithm using benchmarks (e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1)." Using logarithms that have an exact value to approximate the value of other logarithms is one way to demonstrate understanding of estimating and increase understanding of logarithms.

Also included in this curriculum is information pertaining to how the Pre-calculus 30 course connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

In addition, teachers will find discussions about the critical characteristics of mathematics education, inquiry in mathematics, and assessment and evaluation of student learning in mathematics.

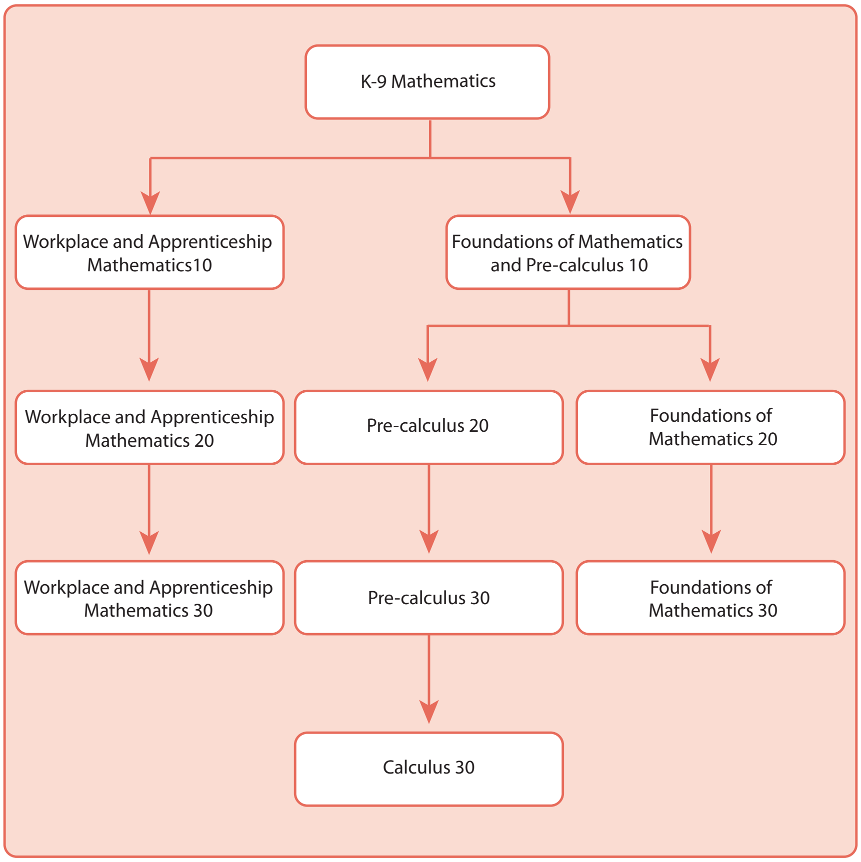
Grades 10-12 Mathematics Framework

Saskatchewan's grades 10 to 12 mathematics curricula are based upon the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10 - 12 Mathematics* (2008). This framework was developed in response to data collected from WNCP post-secondary institutions and business and industry sectors regarding the mathematics needed by students for different disciplines, areas of study, and work areas. From these data there emerged groupings of areas which required the same types of mathematics. Each grouping also required distinct mathematics, so that even if the same topic was needed in more than one of the groupings, it needed to be addressed in different ways.

The result was the creation of a set of pathways consisting of a single grade 10, 11, and 12 course for each of these groupings that were named Workplace and Apprenticeship Mathematics, Pre-calculus, and Foundations of Mathematics. During the defining of the content for these pathways and courses, it became evident that the content for Grade 10 Foundations of Mathematics and Grade 10 Pre-calculus was very similar. The result is the merging of the two Grade 10 courses

(Foundations of Mathematics and Pre-calculus) into a single course entitled Foundations of Mathematics and Pre-calculus 10. The chart below visually illustrates the courses in each pathway and their relationship to each other.

10-12 Mathematics Pathway Framework



No arrows connect courses in different pathways because the content is different between the pathways. Therefore, students wishing to change pathways need to take the prerequisite courses for the pathway. For example, if students are in or have taken Pre-calculus 20, they cannot move directly into either Foundations of Mathematics 30 or Workplace and Apprenticeship Mathematics 30. In addition, if students have not taken Workplace and Apprenticeship Mathematics 10, they must do so before entering into Workplace and Apprenticeship Mathematics 20.

Each course in each pathway is to be taught and learned to the same level of rigour. No pathway or course is considered "easy math"; rather, all pathways and courses present "different maths" for different purposes.

Students may take courses from more than one pathway for credit. The current credit requirements for graduation from grade 12 are one 10 level credit and one 20 level credit in mathematics.

The Ministry of Education recommends that grade 10 students take both grade 10 courses to expose them to the mathematics in each pathway. This also will make transitions easier for those students who wish to change pathways partway through their high school years.

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (2011) on the Ministry of Education website. For additional information related to the components and initiatives of Core Curriculum, please refer to the Ministry website for various policy and foundation documents.

Broad Areas of Learning

Three Broad Areas of Learning reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve understandings, skills, and attitudes related to these Broad Areas of Learning.

Lifelong Learners

Related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Positive Lifestyle*

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that supports their learning of new mathematical concepts and applications that they may encounter within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities, having developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

For mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 mathematics). Students must discover the mathematical knowledge outlined in the curriculum as opposed to the teacher simply covering it.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

(NCTM, 2000, p. 20)

Sense of Self, Community, and Place

Related to the following Goals of Education:

- *Understanding and Relating to Others*
- *Self-Concept Development*
- *Spiritual Development*

To learn mathematics with deep understanding, students need to interact no only with the mathematical content but also with each other. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue and reflection will be exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics provides many opportunities for students to enter into communities beyond the classroom through engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together.

Engaged Citizens

Related to the following Goals of Education:

- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students, such as trends in climate change, homelessness, health issues (e.g., hearing loss, carpal tunnel syndrome, diabetes), and discrimination engage students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for, and understanding of, differing opinions and possible options. With

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

these understandings, students can make better informed and more personalized decisions regarding their roles within, and contributions to, the various communities in which they are members.

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes that are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

K-12 Goals for Developing Thinking:

- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

Related to CEL of Critical and Creative Thinking.

Within their study of mathematics, students must be engaged in personal construction and understanding of mathematical knowledge. This occurs most effectively through student engagement in inquiry and problem solving when they are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which they consider questions such as "What would happen if ...", "Could we find ...", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool that can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

K-12 Goals for Developing Identity and Interdependence:

- *Understanding, valuing, and caring for oneself (intellectually, emotionally, physically, spiritually)*
- *Understanding, valuing, and caring for others*
- *Understanding and valuing social, economic, and environmental interdependence and sustainability.*

Related to CELs of Personal and Social Development and Technological Literacy.

Given an appropriate learning environment in mathematics, students can develop both self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It also can help students take an active role in defining and maintaining the classroom environment and accepting responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning supports students in behaving respectfully towards themselves and others.

Developing Literacies

Through their mathematical learning experiences, students should be engaged in developing their understanding of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be engaged regularly in exploring a variety of representations for mathematical concepts and expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language are to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. Moreover, students should be aware of, and able to make, the appropriate use of technology in mathematics and mathematics learning.

Encourage students to use a variety of forms of representation (concrete manipulatives; physical movement; oral, written, visual, and other symbolic forms) when exploring mathematical ideas, solving problems, and communicating understandings is important. All too often, symbolic representation is assumed to be the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper their understanding becomes.

K-12 Goals for Developing Literacies:

- *Constructing knowledge related to various literacies*
- *Exploring and interpreting the world through various literacies*
- *Expressing understanding and communicating meaning using various literacies.*

Related to CELs of Communication, Numeracy, Technological Literacy, and Independent Learning.

Developing Social Responsibility

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment constructed by the teacher and students supports respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, they become engaged in understanding the situation, concern, or issue, and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for contextual validity, and strengthened.

K-12 Goals for Developing Social Responsibility:

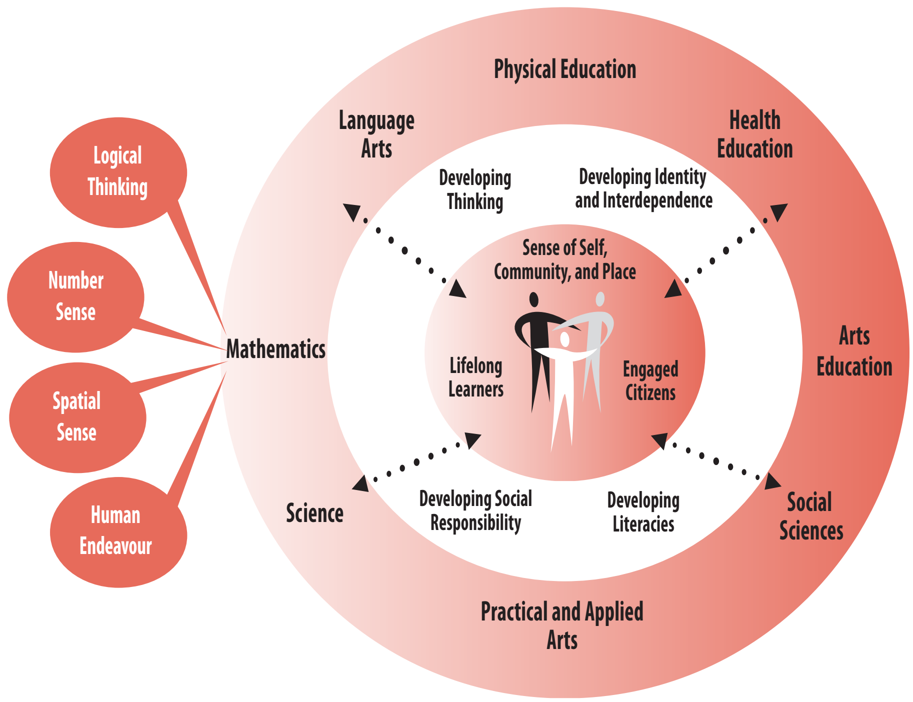
- *Using moral reasoning processes*
- *Engaging in communitarian thinking and dialogue*
- *Taking social action.*

Related to CELs of Communication, Critical and Creative Thinking, Personal and Social Development, and Independent Learning.

K-12 Aim and Goals of Mathematics

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

Defined below are four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are related directly to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes, therefore, also must promote student achievement with respect to the K-12 goals.



Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observing
- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving

- exploring, identifying, and describing relationships
- modelling and representing (including concrete, oral, physical, pictorial, and other symbolic representations)
- conjecturing and asking "what if" (mathematical play).

To develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers, and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing numbers
- relating different operations to each other
- modelling and representing numbers and operations (including concrete, oral, physical, pictorial, and other symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, for students to become flexible and confident in their calculation abilities, and to be able to transfer those abilities to more abstract contexts, students first must develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

The ability to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation and testing of conjectures based upon discovered patterns should drive the students' development of spatial sense, with formulas and definitions resulting from their mathematical learnings.

Mathematics as a Human Endeavour

Meaning does not reside in tools; it is constructed by students as they use tools.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Hunman, 1997, p. 10)

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics

- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematical and personal)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet its particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments, and those developments are determined by the contexts and needs of the time, place, and people.

All students benefit from mathematics learning that values and respects different ways of knowing mathematics and its relationship to the world. The mathematics content found within this curriculum often is viewed in schools and schooling through a Western or European lens, but there are many different lenses, such as those of many First Nations and Métis peoples, through which mathematics can be viewed and understood. The more exposure that all students have to differing ways of understanding and knowing mathematics, the stronger they will become in their number sense, spatial sense, and logical thinking.

The content found within the grade level outcomes for the K-12 mathematics program and its applications first and foremost, is the vehicle through which students can achieve the four K-12 goals of mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula, there is very little to cover, but an awful lot to uncover [discover]". This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience reveal a complex, interrelated set of characteristics that teachers need to be aware of to provide an effective mathematics program.

Assumptions in this Curriculum

The question in mathematics often arises as to whether students should work with fractions, decimals, or both, and if working with fractions, whether mixed numbers or improper fractions should be used. For the purposes of this document, we assume the following:

- If a question or problem is stated with fractions (decimals), the solution should involve fractions (decimals), unless otherwise stated.
- Final fraction solutions can be stated in mixed numbers or improper fractions as long as this is consistent with the original stating of the question or problem.
- The word "or" is used to indicate that students should be able to work with the list of strategies, representations, or approaches given in the list, but they should not be expected to apply more than one of such strategies, representations, or approaches to any given situation or question. For example, in the indicator, "Sketch, with or without the use of technology, the graph of a polynomial function", students should not be expected to explore and utilize both options for sketching a graph.

When engaging in activities related to graphing, the word "sketch" indicates that the graph can be produced without the use of specific tools or an emphasis on precision. The word "draw" indicates that specific tools (such as graphing software or graph paper) should be used to produce a graph of greater accuracy.

Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from "covering" to supporting students in "discovering" mathematical concepts. These facets include:

- the organization of the outcomes
- the seven mathematical processes
- the difference between covering and discovering mathematics

- the development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiquing statements
- the concrete to abstract continuum
- modelling and making connections
- the role of homework
- the importance of ongoing feedback and reflection.

Organization of Outcomes

The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are a succinct set of high-level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses.

For ease of reference, the outcomes in this curriculum are numbered using the following system: PC30.#, where PC refers to Pre-calculus, 30 indicates the course level, and # is the number of the outcome in the list of outcomes. PC30.1 need not be taught before PC30.13, nor do the outcomes need to be taught in isolation of each other.

Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics, rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Pre-calculus 30 are at the discretion of the teacher.

Mathematical Processes

This Pre-calculus 30 curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing, along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. During planning, teachers should carefully consider those indicated processes as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, prior knowledge, the formal language and symbols of mathematics, and new learning.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology once they have had sufficient experience to develop an understanding of that terminology.

Concrete, pictorial, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.

(Caine & Caine, 1991, p. 5)

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you ...?", "Can you ...?", or "What if ...?", the problem solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

For an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students are given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for and engage in finding a variety of strategies for solving problems empowers them to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Mathematical problem-solving often involves moving backwards and forwards between numerical/ algebraic representations and pictorial representations of the problem.

(Haylock & Cockburn, 2003, p. 203)

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. Meaningful inquiry challenges students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.

(NCTM, 2000, p. 191)

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.

(Armstrong, 1993, p. 10)

Technology [T]

Technology tools contribute to student achievement of a wider range of mathematics outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology should not be used as a replacement for basic understandings and intuition.

(NCTM, 2000, p. 25)

Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. Students should understand and appreciate the appropriate use of technology in a mathematics classroom. They also should learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding rather enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social convention or custom of mathematics. This content includes what the symbol for an operation looks like, mathematical terminology, and conventions regarding the recording of symbols and quantities.

Content that can and should be discovered by students is that which can be constructed by students based on prior mathematical knowledge. This content includes things such as strategies, processes, and rules, as well as the students' current and intuitive understandings of quantity, patterns, and shapes. Any learning in mathematics that is a consequence of the logical structure of mathematics can and should be constructed by students.

For example, when learning in relation to outcome PC30.12,

> Demonstrate understanding of permutations, including the fundamental counting principle.

students can explore, discuss, and analyze authentic situations that they encounter everyday using prior knowledge to make connections and help develop strategies for determining the number of choices or arrangements. Problems that engage students in working with a manageable number of choices will allow them to discover a pattern that can be transferred to arrangements with higher numbers. Posing a question that students can represent with coins, markers, handshakes, diagrams, charts, or graphic organizers creates an environment that allows students to construct a strategy and discover the solution. The teacher needs to cover, once the students have made connections to the material, the notation and terminology of a permutation.

Development of Mathematical Terminology

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that they connect the new terminology with their developing mathematical understanding. Therefore, students first must linguistically engage with new mathematical concepts using words that are already known or that make sense to them.

Teachers should model appropriate conventional vocabulary.

(NCTM, 2000, p. 131)

For example, in outcome PC30.9,

> Demonstrate an understanding of logarithms, including:
> - evaluating logarithms
> - relating logarithms to exponents
> - deriving laws of logarithms
> - solving equations
> - graphing.

a "logarithm" and its applications will be new to students. Rather than providing a textbook definition, students construct understanding by developing the meaning through instructional strategies such as concept attainment. Students may discuss and analyze a sample of exponential and logarithmic graphs to understand the existing relationship and build on the concept of inverses. Once students have come to an agreement, "logarithm" can be introduced as mathematical terminology and published definitions can be critiqued.

To help students develop their working mathematical language, teachers must recognize that many students, including First Nations and Métis, may not recognize a specific term or procedure but may, in fact, have a deep understanding of the mathematical topic.

Many perceived learning difficulties in mathematics are the result of students' cultural and personal ways of knowing not being connected to formal mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. Students should be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

Teachers must recognize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understanding. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught in relation to their schema, cultural and environmental context, or real life experiences.

A first step in the actualization of mathematics from First Nations and Métis perspectives is empowering teachers to understand that mathematics is not acultural. As a result, teachers realize that the traditional Western European ways of teaching mathematics also are culturally biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understanding and mathematical self-confidence and ability through a more holistic and constructivist approach to teaching. Teachers need to pay close attention to those factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

Teachers must recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding, and as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas among cultural groups cannot be assumed to be a direct link. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. In addition, they also allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. As well, ethnomathematics demonstrates the relationship between mathematics and cultural anthropology.

Individually and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to themselves personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of Elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. In doing so, the teacher can identify strengths and deficiencies in students' understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, consider the indicator,

> Critique the statement: "If a relation is not a function, then its inverse also will not be a function".

Students who have explored the concept of inverse functions may agree with this statement. The teacher can encourage students to look for a counter-example, define more clearly what they know about inverses using proof, and further analyze the relationship between relations, functions, and their inverses. Having students critique this statement allows the teacher to determine the level of understanding that students have achieved in terms of relations and functions while challenging them to explore a variety of relations and inverses.

Critiquing statements is an effective way to assess students individually, as a small group, or as a large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher but also engage all of the students in a deeper understanding of the topic.

It is important for students to use representations that are meaningful to them.

(NCTM, 2000, p. 140)

The Concrete to Abstract Continuum

In learning mathematics, students should be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may, at one point, be working abstractly, but when a new idea or context arises, they need to return to a more concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving surface area: What is the surface area of your computer? Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives or pictures), or both.

Models and Connections

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.

(NCTM, 2000, p. 139)

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics most frequently are represented by symbolic models, their meaning and purpose often are found in the concrete, physical, pictorial, and oral models, and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and other symbolic models). In addition, students need to make connections between the different representations. These connections are made by having students move from one type of representation to another (i.e., how could you represent what you've done here using mathematical symbols?) or by having them compare their representations with others in the class. In making these connections, students can reflect upon the mathematical ideas and concepts that are being used in their new models.

Making connections also involves looking for patterns. For example, in outcome PC30.2:

> Demonstrate understanding of the unit circle and its relationship to the six trigonometric ratios for any angle in standard position.

Students already have prior knowledge of trigonometric ratios and the Pythagorean theorem that they can use to develop understanding of the unit circle. Using a point on the terminal arm where it intersects with the unit circle as the radius, students can use right angle triangles to make connections between the sides of the triangle and the angle as the point moves around the unit circle. A tangent line and secant line also can be drawn creating another similar right angle triangle for students to analyze and connect to the trigonometric ratios. Students also can further extend the connection to graphs of the trigonometric functions.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that consolidate new learnings with prior knowledge, explore possible solutions, and apply learning to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of drill varies among different learners. In addition, when used as homework, drill and practice frequently causes frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can help students develop deep understanding in Pre-calculus 30, consider outcome PC30.7:

> Extend understanding of transformations to include functions (given in equation or graph form) in general, including horizontal and vertical translations, and horizontal and vertical stretches.

To start students thinking about transformations, the teacher can assign a selection of artwork containing rotations, reflections, translations, or dilations to analyze. To extend transformations to functions, students could match a series of graphs to their corresponding equations for homework. The next day, in small groups, students could compare and discuss their decisions. Further engagement into the process of inquiry can occur when students justify the teacher's choices.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires both the teacher and students to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understanding. Feedback from students to the teacher provides much needed information to planning for further and future learning.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what students do and

Characteristics of Good Math Homework

- *It is accessible to [students] at many levels.*
- *It is interesting both to [students] and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to, and types of, math homework offered to [students] throughout the year.*

(Adapted from Raphel, 2000, p. 75)

Feedback can take many different forms. Instead of saying, “This is what you need to do,” we can ask questions: “What do you think you need to do? What other strategy choices could you make? Have you thought about ...?”

(Stiff, 2001, p. 70)

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.

(NCTM, 2000, p. 72)

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences .... A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.

(Haylock & Cockburn, 2003, p. 18)

What might you hear or see in a Pre-calculus 30 classroom that would indicate to you that students are developing a deep understanding?

do not know. Through such reflections, not only can a teacher make better informed instructional decisions, but also a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, students must learn by constructing knowledge, with very few ideas relayed directly by the teacher. As an example, the teacher will have to show and name function notation; however, first, the students could explore those ideas important for working with function notation.

Teachers should analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to provide opportunities for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

A mathematics learning environment should include an effective interplay of:

- reflecting
- exploring patterns and relationships
- sharing ideas and problems
- considering different perspectives
- decision making
- generalizing
- verifying and proving
- modelling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. Conversely, when they learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are involved and engaged directly in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning of curriculum content and skills.

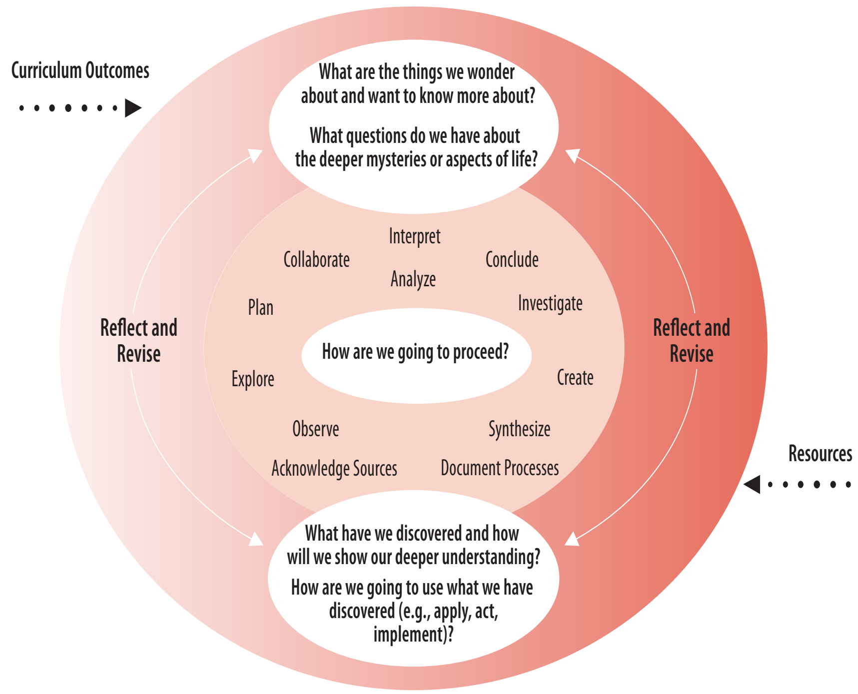
(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process but rather a cyclical one, with parts of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic demonstrates the cyclical inquiry process.

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.

(Mills & Donnelly, 2001, p. xviii)

Constructing Understanding Through Inquiry



Effective questions:

- *cause genuine and relevant inquiry into the important ideas and core content*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answer*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons*
- *spark meaningful connections with prior learning and personal experiences*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as they become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, develop conclusions, document and reflect on learning, and generate new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional similar problems, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation already has been determined, it no longer is problem solving. Students must understand this difference too.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points. However, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment.

Developing questions evoked by students' interests have potential for rich and deep learning. Compelling questions initiate and guide the inquiry, and give students direction for discovering deep understanding about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When or why might you want to use the fundamental counting principle?"
- "How do you know when you have an answer?"
- "Will this strategy work for all situations?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning. Questioning also should be used to encourage students to reflect on the inquiry process and on the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- help students make sense of the mathematics.
- are open-ended, whether in answer or approach, as there may be multiple answers or multiple approaches.
- empower students to unravel their misconceptions.
- not only require the application of facts and procedures but also encourage students to make connections and generalizations.
- are accessible to all students and offer an entry point for all students.
- lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.

(Schuster & Canavan Anderson, 2005, p. 3)

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.

(Schuster & Canavan Anderson, 2005, p. 1)

Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics can take the form of reflective journals, notes, models, works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understanding.

Students must engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen their understanding.

Outcomes and Indicators

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.1 Extend understanding of angles to angles in standard position, expressed in degrees and radians. **[CN, ME, R, V]**	a. Sketch angles in standard position including positive and negative degrees. b. Investigate and describe the relationship between different systems of angle measurements, with emphasis on radians and degrees. c. Sketch, in standard position, an angle measuring 1 radian. d. Sketch, in standard position, any angle measuring $k\pi$ radians where $k \in Q$. e. Develop and apply strategies for converting between angle measures in degrees and radians (exact value or decimal approximation). f. Develop and apply strategies for determining all angles that are coterminal to an angle within a specified domain (in degrees and radians). g. Develop, explain, and apply strategies for determining the general form for all angles that are coterminal to a given angle (in degrees and radians). h. Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius r, and solve situational questions based on that relationship.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.2 Demonstrate understanding of the unit circle and its relationship to the six trigonometric ratios for any angle in standard position. **[CN, ME, PS, R, T, V]**	a. Derive the equation of a circle with centre (0,0) and radius *r*. b. Derive the equation of the unit circle from the application of the Pythagorean theorem or the distance formula. c. Develop and generalize the six trigonometric ratios in terms of *x*, *y*, and *r*, using a point that is the intersection of the terminal arm of an angle with the unit circle. d. Develop, generalize, and apply strategies for determining the six trigonometric ratios for any angle given a point on the terminal arm of the angle. e. Determine, with technology, the approximate value of the trigonometric ratios for any angle (in radians or degrees).

Outcomes	Indicators
PC30.2 continued	f. Develop, generalize, explain, and apply strategies, including using the unit circle or a reference triangle, for determining the exact trigonometric ratios for angles whose measures are multiples of 0°, 30°, 45°, 60°, 90° (when expressed in degrees), 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, or $\frac{\pi}{2}$ (when expressed in radians). g. Explain and apply strategies (with or without the use of technology) to determine the measures, in degrees or radians, of the angles in a specified domain that have a particular trigonometric ratio value. h. Explain and apply strategies to determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain. i. Sketch a diagram to represent the context of a problem that involves trigonometric ratios. j. Solve situational questions using trigonometric ratios.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.3 Demonstrate understanding of the graphs of the primary trigonometric functions. **[CN, PS, T, V]**	a. Sketch, with or without technology, the graph of y=sin*x*, y =cos*x*, and y=tan*x*. b. Determine and summarize the characteristics (amplitude, asymptotes, domain, period, range, and zeros) of the graphs of *y* = sin*x*, *y* = cos*x*, or *y* = tan*x*. c. Develop, generalize, and explain strategies for determining the transformational impact of changing the coefficients a, b, c, and d in $y = a\sin b(x - c) + d$ and $y = a\cos b(x - c) + d$ on the graph of *y* = sin*x* and *y* = cos*x* respectively, including amplitude, asymptotes, domain, period, phase shift, range, and zeros. d. Develop and apply strategies to sketch, without technology, graphs of the form $y = a\sin b(x - c) + d$ or $y = a\cos b(x - c) + d$. e. Write equations for given graphs of sine or cosine functions. f. Identify, with justification, a trigonometric function that models a situational question. g. Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a situational question. h. Solve situational questions by analyzing the graph of trigonometric functions.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour	
Outcomes	**Indicators**
PC30.4 Demonstrate understanding of first- and second-degree trigonometric equations. **[CN, PS, R, T, V]**	a. Verify, with or without technology, whether or not a value is a solution to a particular trigonometric equation. b. Develop and apply strategies for determining algebraically the exact form of the solution to a trigonometric equation. c. Determine, using technology, the approximate solution in degrees and radians of a trigonometric equation in a restricted domain. d. Explain the relationship between the general solution of trigonometric equations to the zeros of the related trigonometric functions limited to sine and cosine functions. e. Determine, using technology, the general solutions for trigonometric equations. f. Analyze solutions for given trigonometric equations to identify errors, and correct if necessary.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour	
Outcomes	**Indicators**
PC30.5 Demonstrate understanding of trigonometric identities including: • **reciprocal identities** • **quotient identities** • **Pythagorean identities** • **sum or difference identities (restricted to sine, cosine, and tangent)** • **double-angle identities (restricted to sine, cosine, and tangent)** **[R, T, V]**	a. Explain the difference between a trigonometric identity and a trigonometric equation. b. Verify numerically (using degrees or radians) whether or not a trigonometric statement is a trigonometric identity. c. Critique statements such as "If three different values verify a trigonometric identity, then the identity is valid". d. Determine, with the use of graphing technology, the potential validity of a trigonometric identity. e. Determine the non-permissible values of a trigonometric identity. f. Develop, explain, and apply strategies for proving trigonometric identities algebraically. g. Explain and apply strategies for determining the exact value of a trigonometric ratio by using sum, difference, and double-angle identities.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.6 Demonstrate an understanding of operations on, and compositions of, functions. **[CN, R, T, V]**	a. Sketch the graph of a function that is the sum, difference, product, or quotient of two functions whose graphs are given. b. Write the equation of a function that results from the sum, difference, product, or quotient of two or more functions. c. Develop, generalize, explain, and apply strategies for determining the domain and range of a function that is the sum, difference, product, or quotient of two other functions. d. Write a function as the sum, difference, product, or quotient (or some combination thereof) of two or more functions. e. Develop, generalize, explain, and apply strategies for determining the composition of two functions: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$. f. Develop, generalize, explain, and apply strategies for evaluating a composition of functions at a particular point. g. Develop, generalize, explain, and apply strategies for sketching the graph of composite functions in the form: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$ where the equations or graphs of $f(x)$ and $g(x)$ are given. h. Write a function as a composition of two or more functions. i. Write a function by combining two or more functions through operations on, and compositions of, functions.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.7 Extend understanding of transformations to include functions (given in equation or graph form) in general, including horizontal and vertical translations, and horizontal and vertical stretches. [C, CN, R, V]	a. Compare and analyze various graphs of transformations of the function $y = f(x)$, and generalize about the effect of the placement of different coefficients on the original graph of $y = f(x)$. b. Develop, generalize, explain, and apply strategies for sketching transformations of the graph of $y = f(x)$ to give the graph of $y - k = af(b(x - h))$. c. Write the equation of a function that has undergone specified vertical translations, horizontal translations, vertical stretches, and/or horizontal translations of the function $y = f(x)$ for which the equation is given.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.8 Demonstrate understanding of functions, relations, inverses and their related equations resulting from reflections through the: • **x-axis** • **y-axis** • **line y = x.** [C, CN, R, V]	a. Generalize and apply the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x-axis, the y-axis, or the line $y = x$. b. Develop and apply strategies for sketching the reflection of a function $y = f(x)$ through the x-axis, the y-axis, or the line $y = x$ when the graph of $f(x)$ is given but the equation is not. c. Develop and apply strategies for sketching the graphs of $y = -f(x)$, $y = f(-x)$, and $x = -f(y)$ when the graph of $f(x)$ is given and the equation is not. d. Develop and apply strategies for writing the equation of a function that is the reflection of the function $f(x)$ through the x-axis, y-axis, or line $y = x$. e. Develop and apply strategies for sketching the inverse of a relation, including reflection across the line $y = x$ and the transformation $(x, y) \Rightarrow (y, x)$. f. Sketch the graph of the inverse relation, given the graph of the relation. g. Develop, generalize, explain, and apply strategies for determining if one or both of a relation and its inverse are functions. h. Determine what restrictions must be placed on the domain of a function for its inverse to be a function.

Outcomes

PC30.8 continued

Indicators

i. Critique statements such as "If a relation is not a function, then its inverse also will not be a function".

j. Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation.

k. Explain the relationship between the domains and ranges of a relation and its inverse.

l. Develop and apply numeric, algebraic, and graphic strategies to determine if two relations are inverses of each other.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes

PC30.9 Demonstrate an understanding of logarithms including:

- **evaluating logarithms**
- **relating logarithms to exponents**
- **deriving laws of logarithms**
- **solving equations**
- **graphing.**

[C, CN, ME, PS, R, T, V]

Indicators

a. Explain the relationship between powers, exponentials, logarithms, and radicals.

b. Express a logarithmic expression as an exponential expression and vice versa.

c. Determine, without technology, the exact value of a logarithm such as $\log_2 8$.

d. Explain how to estimate the value of a logarithm using benchmarks (e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1).

e. Derive and explain the laws of logarithms.

f. Apply the laws of logarithms to determine equivalent expressions for given logarithmic statements.

g. Determine, using technology, the approximate value of a logarithmic expression (e.g., $\log_2 9$).

h. Solve exponential equations in which the bases are powers of one another.

i. Solve exponential equations in which the bases are not powers of one another.

j. Develop, generalize, explain, and apply strategies for solving logarithmic equations and verify the solutions.

k. Explain why a value obtained in solving a logarithmic equation may be extraneous.

l. Solve situational questions that involve exponential growth or decay, such as loans, mortgages, and investments.

Outcomes	Indicators
PC30.9 continued	m. Solve situational questions involving logarithmic scales, such as the Richter scale and pH scale. n. Analyze graphs of exponential functions of the form $y = a^x, a > 0$ and report about the relationships between the value of *a* and the domain, range, horizontal asymptote, and intercepts. o. Sketch, with or without the use of technology, the graphs of exponential functions of the form $y = a^x, a > 0$. p. Explain the role of the horizontal asymptote for exponential functions. q. Develop, generalize, explain, and apply strategies for sketching transformations of the graph of $y = a^x, a > 0$. r. Analyze graphs of logarithmic functions of the form $y = \log_b x, b > 1$ and report about the relationships between the value of b and the domain, range, vertical asymptote, and intercepts. s. Sketch, with or without technology, the graphs of logarithmic functions of the form $y = \log_b x, b > 1$. t. Explain the role of the vertical asymptote for logarithm functions. u. Develop, generalize, explain, and apply strategies for sketching transformations of the graph of $y = \log_b x, b > 1$. v. Demonstrate graphically that $y = \log_b x, b > 1$ and $y = b^x, b > 0$ are inverses of each other.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.10 Demonstrate understanding of polynomials and polynomial functions of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). **[C, CN, ME, T, V]**	a. Develop, generalize, explain, and apply long division for dividing polynomials by binomials of the form $x - a, a \in I$. b. Compare long division of polynomial expressions by binomial expressions to synthetic division, and explain why synthetic division works. c. Divide a polynomial expression by a binomial expression of the form $x - a, a \in I$ using synthetic division. d. Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.

Outcomes	Indicators
PC30.10 continued	e. Generalize, through inductive reasoning, the relationship between the remainder when a polynomial expression is divided by $x - a, a \in \mathrm{I}$ and the value of the polynomial expression at $x = a$ (The Remainder Theorem). f. Explain and apply the factor theorem to express a polynomial expression as a product of factors. g. Categorize, with justification, a set of functions into polynomial functions and non-polynomial functions. h. Analyze graphs of polynomial functions to determine the impact of changing the values of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function. i. Generalize and apply strategies for graphing polynomial functions of an odd or even degree. j. Explain the relationship between: • the zeros of a polynomial function • the roots of the corresponding polynomial equation • the x-intercepts of the graph of the polynomial function. k. Explain and apply strategies for determining the behaviour of the graph of a polynomial function at zeros with different multiplicities. l. Sketch, with or without the use of technology, the graph of a polynomial function. m. Solve situational questions by modelling the situations with polynomial functions and analyzing the graphs of the functions.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.11 Demonstrate understanding of radical and rational functions with restrictions on the domain. **[CN, R, T, V]**	a. Sketch the graph of the function $y = \sqrt{x}$ using a table of values, and state the domain and range of the function. b. Develop, generalize, explain, and apply transformations to the function $y = \sqrt{x}$ to sketch the graph of $y - k = a\sqrt{b(x-h)}$. c. Sketch the graph of the function $y = \sqrt{f(x)}$ given the graph of the function $y = f(x)$, and compare the domains and ranges of the two functions.

Outcomes	Indicators
PC30.11 continued	d. Describe the relationship between the roots of a radical equation and the x-intercepts of the graph of the corresponding radical function. e. Determine, graphically, the approximate solutions to radical equations. f. Sketch rational functions, with and without the use of technology. g. Explain the behaviour (shape and location) of the graphs of rational functions for values of the dependent variable close to the location of a vertical asymptote. h. Analyze the equation of a rational function to determine where the graph of the rational function has an asymptote or a hole, and explain why. i. Match a set of equations for rational and radical functions to their corresponding graphs. j. Describe the relationship between the roots of a rational equation and the x-intercepts of the graph of the corresponding rational function. k. Determine graphically an approximate solution to a rational equation. l. Critique statements such as "Any value that makes the denominator of a rational function equal to zero will result in a vertical asymptote on the graph of the rational function".

Goals: Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.12 Demonstrate understanding of permutations, including the fundamental counting principle. **[C, PS, R, V]**	a. Develop and apply strategies, such as lists or tree diagrams, to determine the total number of choices or arrangements possible in a situation. b. Explain why the total number of possible choices is found by multiplying rather than adding the number of ways that individual choices can be made. c. Provide examples of situations relevant to self, family, and community where the fundamental counting principle can be applied to determine the number of possible choices or arrangements. d. Create and solve situational questions that involve the application of the fundamental counting principle.

Outcomes	Indicators
PC30.12 continued	e. Count, using graphic organizers, the number of ways to arrange the elements of a set in a row. f. Develop, generalize, explain, and apply strategies, including the use of factorial notation, to determine the number of permutations possible if *n* different elements are taken *n* or *r* at a time. g. Explain why *n* must be greater than or equal to *r* in the notation ${}_nP_r$. h. Solve equations that involve ${}_nP_r$ notation such as ${}_nP_2 = 30$. i. Develop, generalize, explain, and apply strategies for determining the number of permutations possible when two or more elements in the set are identical (non-distinguishable).

Goals: Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
PC30.13 Demonstrate understanding of combinations of elements, including the application to the binomial theorem. **[C, CN, PS, R, V]**	a. Explain, with examples, how to distinguish between situations that involve permutations and those that involve combinations. b. Develop, generalize, explain, and apply strategies for determining the number of ways that a subset of *k* can be selected from a set of *n* different elements. c. Develop, generalize, explain, and apply strategies to determine combinations of *n* different elements taken *r* at a time in situational questions. d. Explain why *n* must be greater than or equal to *r* in the notation ${}_nC_r$ or $\binom{n}{r}$. e. Prove or explain using examples ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$. f. Solve equations involving combinations (e.g., ${}_nC_2 = 15$ or $\binom{n}{2} = 15$). g. Explore and describe patterns found within Pascal's triangle, including the relationship between consecutive rows. h. Explore and describe the relationship between the coefficients of the terms in $(x+y)^n$, and the combinations. i. Develop, generalize, explain, and apply strategies for expanding $(x+y)^n, n \le 4$. j. Develop, generalize, explain, and apply strategies for determining specific terms within a particular expansion of $(x+y)^n$ given $n \in \mathrm{N}$.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement is based on the outcomes in the provincial curriculum.

Assembling evidence from a variety of sources is more likely to yield an accurate picture.

(NCTM, 2000, p. 24)

Assessment involves the systematic collection of information about student learning with respect to:

- achievement of provincial curriculum outcomes
- effectiveness of teaching strategies employed
- student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information unrelated to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Pre-calculus 30. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment should not merely be done to students; rather it should be done for students.

(NCTM, 2000, p. 22)

Assessment as learning involves student reflection and monitoring of her/his progress related to curricular outcomes, and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

What are examples of assessments as learning that could be used in Pre-calculus 30 and what would be the purpose of those assessments?

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement, and:

- provides an opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

Assessment should become a routine part of the ongoing classroom activity rather than an interruption.

(NCTM, 2000, p. 23)

In mathematics, students need to be engaged regularly in assessment as learning. The various types of assessments should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learning as well as opportunities for them to set and assess personal learning goals related to the content of Pre-calculus 30.

Glossary

Compositions of functions: A process through which an entire function is substituted into another function; that is, the output from one function becomes the input for the next function. In math terms, the range (the y-value answers) of one function becomes the domain (the x-values) of the next function.

Coterminal angles: Two angles are coterminal if they are drawn in the standard position and both have their terminal sides in the same location. For example, 50° and 410° are coterminal angles.

Inverse of a function: The inverse of a function has all the same points as the original function, except the domain (the x's) and the range (the y's) have traded places. All elements of the domain become the range, and all elements of the range become the domain. If $f(g(x)) = x$ and $g(f(x)) = x$, then the functions f and g are inverses. Conversely, if f and g are inverses, then $f(g(x)) = x$ and $g(f(x)) = x$. This denotes an "if and only if" sentence. For instance, if a function is made up of points: { (1,0), (-3,5), (0,4) }, then the inverse is the set of points: {(0,1), (5,-3), (4,0) }.

Non-permissible value: A non-permissible value for a variable in an expression that results in a denominator of 0. This implies division by 0 and thus is undefined, so those values for the variable(s) must be excluded.

Situational questions: Mathematical questions that are asked within the context of a particular situation. Situational questions may be actual problems (something that the student does not yet know how to solve) or practice (something that the student has seen examples of how to solve).

Trigonometric identity: Name given to certain relationships that exist between various trigonometry ratios. These equalities involve trigonometric functions and are true for every single value of the occurring variables. These relationships are useful whenever expressions involving trigonometric functions need to be simplified.

Zeros of a function: A value of x of the domain of a function $f(x)$ for which $f(x) = 0$ or $f(x)$ vanishes at 0. For example, 1 and -1 are zeros of $f(x) = x^2 - 1$ because for both 1 and -1 $f(x) = 0$ and both 3 and 1 are zeros of $f(x) = x^2 - 4x + 3$ because $3^2 - 4(3) + 3$ and $1^2 - 4(1) + 3 = 0$.

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Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Pre-calculus 30 Curriculum

1. Please indicate your role in the learning community

☐ parent ☐ teacher ☐ resource teacher

☐ guidance counsellor ☐ school administrator ☐ school board trustee

☐ teacher librarian ☐ school community council member

☐ other ____

What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

☐ print

☐ online

b) Please indicate which format(s) of the curriculum you prefer:

☐ print

☐ online

3. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
appropriate for its intended purpose	1	2	3	4
suitable for your use	1	2	3	4
clear and well organized	1	2	3	4
visually appealing	1	2	3	4
informative	1	2	3	4

4. Explain which aspects you found to be:

Most useful:

Least useful:

5. Additional comments:

6. Optional:

Name: ____

School: ____

Phone: ________________________ Fax: ________________________

Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

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