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Saskatchewan Curriculum

Pre-calculus

20

Ministry of
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Introduction

Pre-calculus 20 is to be allocated 100 hours. It is important for students to receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

The outcomes in the Pre-calculus 20 course are based upon the students' prior learning and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These learning experiences prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts. The outcomes in this course are the prerequisite outcomes for Pre-calculus 30.

The outcomes in this curriculum define content that is considered a high priority in fields of study and areas of work for which the Pre-calculus pathways is required. The outcomes represent the ways of thinking or behaving like a mathematics discipline area expert in those fields of study or areas of work.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to understand and/or be able to do in order to achieve the learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things students should understand or be able to do if they have attained an outcome.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Pre-calculus 20 outcomes are based upon the renewed Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for 10-12 Mathematics (2008)* .

Within the outcomes and indicators in this curriculum, the terms "including" and "such as", as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, consider the indicator "Generalize a rule from sets of graphs, using inductive reasoning, and explain how different values of a (including 1, 0, and -1) transform the graph of $y = ax^2$". It is expected that along with exploring any other values for a, the values of 1, 0, and -1 will definitely be included.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, consider the indicator "Develop, generalize, explain, and apply strategies, such as case analysis, graphing, roots and test points, or sign analysis, to solve one-variable quadratic inequalities". This list, although not exhaustive, gives a number of possible types of strategies that students could derive and use in the solving of one-variable quadratic inequalities. Students are not required to derive or use any of these strategies in particular.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, consider the indicator "Analyze, describe, and generalize the relationship between the reference angles for angles (in standard positions) that are reflections of each other across both the x- and y- axes (e.g., 30° and 150°, or -60° and 60°)". These examples are provided to demonstrate what is meant by reflections across the x-axis or y-axis. It is neither an exhaustive list of all such possible angle pairs, nor is it a mandatory list.

Also included in this curriculum is information regarding how the Pre-calculus 20 course connects to the K-12 goals for mathematics. These goals (page 8) define the purpose of mathematics education for Saskatchewan students.

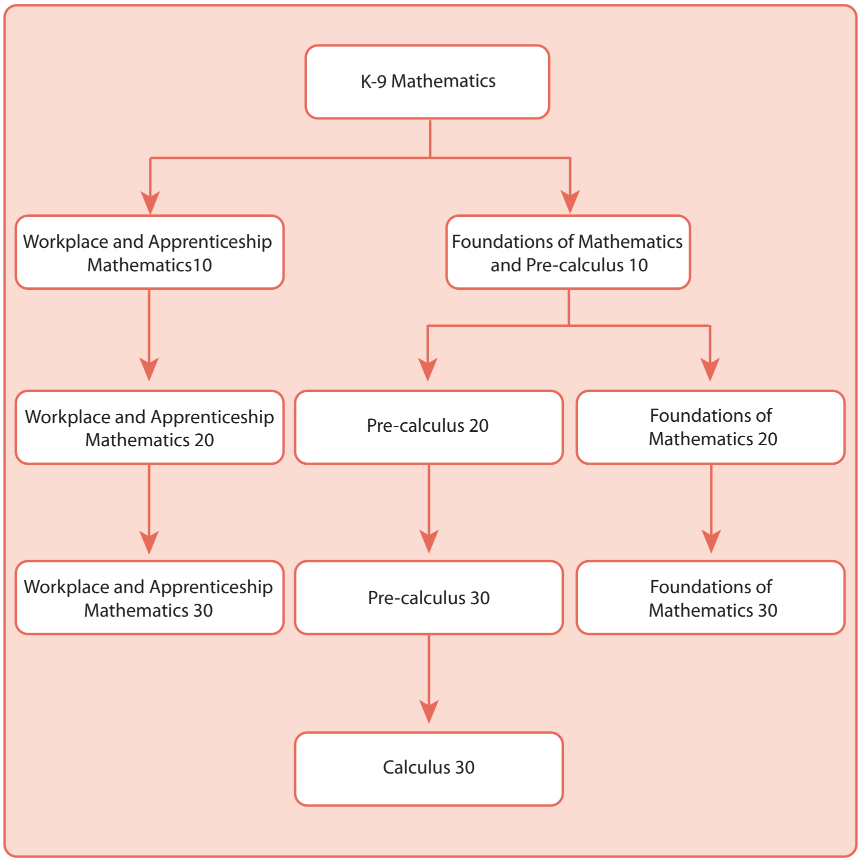
In addition, teachers will find discussions of the critical characteristics of mathematics education, inquiry in mathematics, and assessment and evaluation of student learning in mathematics in this document.

Grades 10 - 12 Mathematics Framework

Saskatchewan's grades 10 to 12 mathematics curricula are based upon the Western and Northern Canadian Protocol's *(WNCP) The Common Curriculum Framework for Grades 10 - 12 Mathematics* (2008). This framework was developed in response to data collected from post-secondary institutions and business and industry sectors regarding the mathematics needed by students for different disciplines, areas of study, and work areas. From these data, there emerged groupings of areas which required the same types of mathematics. Each grouping also required distinct mathematics, so that even if the same topic was needed in more than one of the groupings, it needed to be addressed in different ways.

The result was the creation of a set of pathways consisting of a single grade 10, 11, and 12 course for each of these groups which were named Workplace and Apprenticeship Mathematics, Pre-calculus, and Foundations of Mathematics. During the defining of the content for these pathways and courses, it became evident that the content for Grade 10 Foundations of Mathematics and Grade 10 Pre-calculus is very similar. The result is the merging of the two Grade 10 courses (Foundations of Mathematics, and Pre-calculus) into a single course entitled Foundations of Mathematics and Pre-calculus 10. The following chart visually illustrates the courses in each pathway and their relationship to each other.

10-12 Mathematics Pathways Framework



It is important to note that there are no arrows connecting courses in different pathways. This is because the content is different between the pathways, so students wishing to change pathways need to first get the prerequisite courses for the pathway. For example, if students are in or have taken Pre-calculus 20, they cannot move directly into either Foundations of Mathematics 30 or Workplace and Apprenticeship Mathematics 30. In addition, if students have not already taken Workplace and Apprenticeship Mathematics 10, they must do so before entering into Workplace and Apprenticeship Mathematics 20.

Each course in each pathway is to be taught and learned to the same level of rigour. No pathway or course is considered "easy math"; rather, all pathways and courses present "different maths" for different purposes.

Students may take courses from more than one pathway for credit. The current credit requirements for graduation from grade 12 are: one 10 level credit and one 20 level credit in mathematics.

The Ministry of Education recommends that grade 10 students take both grade 10 courses to give the students an idea of what the mathematics in each pathway is like. This will also make transitions easier for those students who wish to change pathways partway through their high school years.

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (2009) on the Ministry of Education website. For additional information related to the components and initiatives of Core Curriculum, please refer to the Ministry website (*http://www.education.gov.sk.ca/policy*) for various policy and foundation documents, including the following:

- *Understanding the Common Essential Learnings: A Handbook for Teachers* (1988)
- *Objectives for the Common Essential Learnings (CELs)* (1998)
- *Renewed Objectives for the Common Essential Learnings of Critical and Creative Thinking (CCT) and Personal and Social Development (PSD)* (2008)
- *The Adaptive Dimension in Core Curriculum* (1992)
- *Policy and Procedures for Locally-developed Courses of Study* (2010)
- *Connections: Policy and Guidelines for School Libraries in Saskatchewan* (2008)
- *Diverse Voices: Selecting Equitable Resources for Indian and Métis Education* (2005)
- *Gender Equity: Policies and Guidelines for Implementation* (1991)
- *Instructional Approaches: A Framework for Professional Practice* (1991)
- *Multicultural Education and Heritage Language Education Policies* (1994)
- *Physical Education: Safety Guidelines for Policy Development* (1998)
- *Classroom Curriculum Connections: A Teacher's Handbook for Personal-Professional Growth* (2001).

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Lifelong Learners

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

Related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Positive Lifestyle*

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

(NCTM, 2000, p. 20)

Sense of Self, Community, and Place

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue and reflection are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Related to the following Goals of Education:

- *Understanding and Relating to Others*
- *Self Concept Development*
- *Spiritual Development*

Mathematics provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together.

Related to the following Goals of Education:

- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students, such as trends in climate change, homelessness, health issues (e.g., hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school , community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

K-12 Goals for Developing Thinking:

- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

Related to CEL of Critical and Creative Thinking.

Developing Thinking

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This occurs most effectively through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if ...", "Could we find ...", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accepting responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

K-12 Goals for Developing Identity and Interdependence:

- *Understanding, valuing, and caring for oneself (intellectually, emotionally, physically, spiritually)*
- *Understanding, valuing, and caring for others*
- *Understanding and valuing social, economic, and environmental interdependence and sustainability.*

Related to CELs of Personal and Social Development and Technological Literacy.

Developing Literacies

Through their mathematical learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language are to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings.

All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

K-12 Goals for Developing Literacies:

- *Constructing knowledge related to various literacies*
- *Exploring and interpreting the world through various literacies*
- *Expressing understanding and communicating meaning using various literacies.*

Related to CELs of Communication, Numeracy, Technological Literacy, and Independent Learning.

Developing Social Responsibility

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the

K-12 Goals for Developing Social Responsibility:

- *Using moral reasoning processes*
- *Engaging in communitarian thinking and dialogue*
- *Taking social action.*

Related to CELs of Communication, Critical and Creative Thinking, Personal and Social Development, and Independent Learning.

learning environment constructed by the teacher and students supports respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for contextual validity, and strengthened.

K-12 Aim and Goals of Mathematics

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

Defined below are four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the K-12 goals.

Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observing
- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns

- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolic representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and other symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to be able to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation and testing of conjectures based upon patterns that are discovered should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics

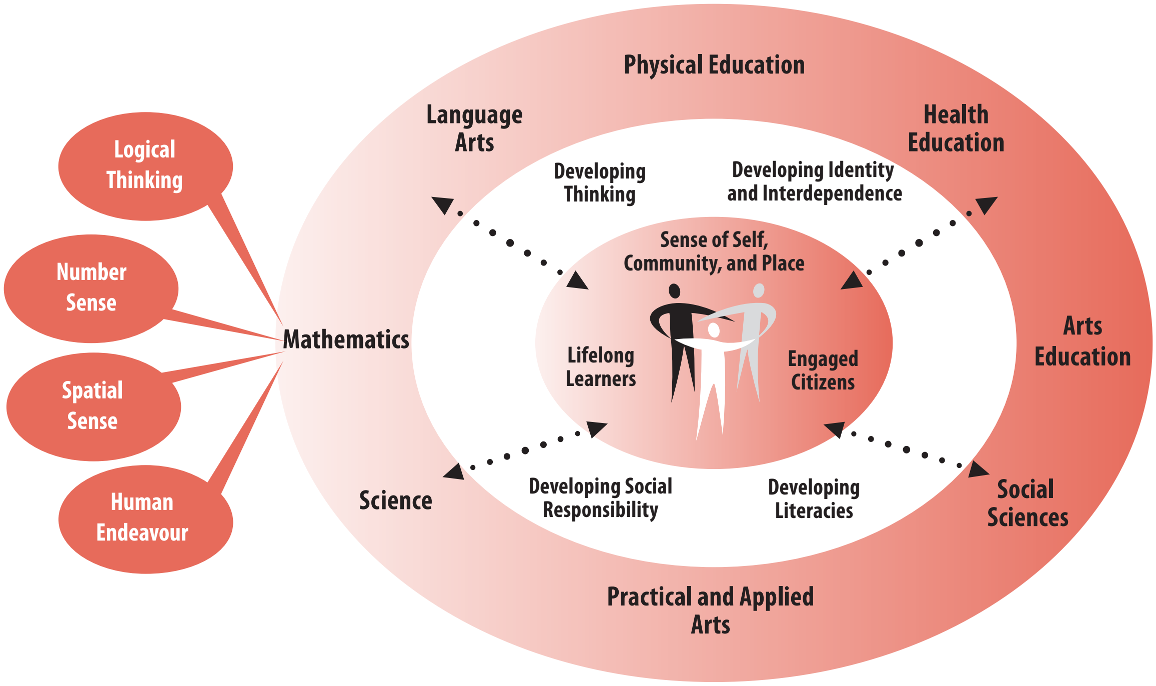
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematical and personal)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet its particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

All students benefit from mathematics learning which values and respects different ways of knowing mathematics and its relationship to the world. The mathematics content found within this curriculum is often viewed in schools and schooling through a Western or European lens, but there are many different lenses through which mathematics can be viewed, such as those of many First Nations and Métis peoples, and understood. The more exposure that all students have to differing ways of understanding and knowing mathematics, the stronger students will become in their number sense, spatial sense, and logical thinking.

Meaning does not reside in tools; it is constructed by students as they use tools.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Hunman, 1997, p. 10)



The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four K-12 goals of mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]". This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

Assumptions in this Curriculum

The question in mathematics often arises as to whether students should work with fractions, decimals, or both and, if working with fractions, should mixed numbers or improper fractions be used. For the purposes of this document, we will assume the following:

- If a question or problem is stated with fractions (decimals), the solution should involve fractions (decimals) unless otherwise stated.
- Final fraction solutions can be stated in mixed numbers or improper fractions as long as they are consistent with the original stating of the question or problem.
- The word "or" is used to indicate that students should be able to work with the list of strategies, representations, or approaches given in the list, but they should not be expected to apply more than one of such strategies, representations, or approaches to any given situation or question. For example, in the indicator "Analyze quadratic functions (with or without the use of technology) to answer situational questions", students should not be expected to analyze a quadratic functions both with technology and without it, but they should be able to analyze any quadratic function by either using technology or not.

In addition, this curriculum assumes that when engaging in activities related to graphing, the word "sketch" should be used to indicate that the graph can be produced without the use of specific tools or an emphasis on precision. The word "draw" should be used to indicate that specific tools (such as graphing software or graph paper) should be used to produce a graph of greater accuracy.

Critical Characteristics of Mathematics Education

The following sections, in this curriculum, highlight some of the different facets for teachers to consider in the process of changing from "covering" to supporting students in "discovering" mathematical concepts. These facets include:

- the organization of the outcomes
- the seven mathematical processes
- the difference between covering and discovering mathematics
- the development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiquing statements
- the concrete to abstract continuum
- modelling and making connections
- the role of homework
- the importance of ongoing feedback and reflection.

Organization of Outcomes

The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: P20.#, where P indicates Pre-calculus, 20 indicates a 20 level course, and where # is the number of the outcome in the list of outcomes. It should be noted, for example, that P20.1 need not be taught before P20.11, nor do the outcomes need to be taught in isolation of each other. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Pre-calculus 20 is at the discretion of the teacher.

Mathematical Processes

This Pre-calculus 20 curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing, along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. During planning, teachers should carefully consider those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, prior knowledge, the formal language and symbols of mathematics, and new learning.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding of that terminology.

Concrete, pictorial, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.

(Caine & Caine, 1991, p. 5)

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you ...?", "Can you ...?", or "What if ...?", the problem solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

Mathematical problem-solving often involves moving backwards and forwards between numerical/ algebraic representations and pictorial representations of the problem.

(Haylock & Cockburn, 2003, p. 203)

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.

(NCTM, 2000, p. 191)

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. Meaningful inquiry challenges students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.

(Armstrong, 1993, p. 10)

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Technology should not be used as a replacement for basic understandings and intuition.

(NCTM, 2000, p. 25)

Technology [T]

Technology tools contribute to student achievement of a wider range of mathematics outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols and quantities.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies, processes, and rules, as well as the students' current and intuitive understandings of quantity, patterns, and shapes. Any learning in mathematics that is a consequence of the logical structure of mathematics can and should be constructed by students.

For example, when learning in relation to outcome P20.11:

> Demonstrate understanding of reciprocal functions of:
>
> - linear functions
> - quadratic functions.
>
> [CN, R, T, V]

students need to be engaged in exploring what a reciprocal function is, and to discover patterns in the values of the reciprocals of different y-values from the original linear or quadratic function, rather than being told and/or shown what to do by the teacher. In engaging the students in the development of these understandings, the students are able to relate their new learnings to past learnings, and also expand their abilities related to number sense, spatial sense, logical thinking, and understanding mathematics as a human endeavour (the four goals of K-12 mathematics).

Teachers should model appropriate conventional vocabulary.

(NCTM, 2000, p. 131)

Development of Mathematical Terminology

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that they already know or that make sense to them.

For example, in outcome P20.5:

> Demonstrate understanding of the cosine law and sine law, including the ambiguous case.
>
> [C, CN, PS, R, T]

the students need to first focus on patterns they find and strategies that they can develop in order to solve for missing side lengths and angles in obtuse and acute triangles. Once the students have begun to generalize and verify the relationships and strategies that they are discovering, the teacher should then help the students to derive the traditional cosine and sine laws. At that point, when the laws have meaning and personal significance, they should be named for the students. Prior to this point in their learning, the names "cosine law" and "sine law" are meaningless, and will not be understood by the students.

In helping students develop their working mathematical language, it is also important for the teachers to recognize that for many students, including First Nations and Métis, they may not recognize a specific term or procedure, but may in fact have a deep understanding of the mathematical topic. Many perceived learning difficulties in mathematics are the result of students' cultural and personal ways of knowing not being connected to formal mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. For example, consider the sentence "The shooting of the hunters was terrible" (Paulos, 1980, p. 65). Were the hunters that bad of a shot, was it terrible that the hunters got shot, was it terrible that they were shooting, or is this all about the photographs that were taken by the hunters? It is important that students be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

It is important for teachers to realize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understandings. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught to their schema, cultural and environmental context, or real life experiences. A first step in actualization of mathematics from First Nations and Métis perspectives is to empower teachers to understand that mathematics is not acultural. As a result, teachers realize that the traditional ways of teaching the mathematics are also culturally-biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understandings and mathematical self-confidence and ability through a more holistic and constructivist approach to learning. Teachers need to pay close attention to the factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

It is important for teachers to recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others, as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding and, as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas between cultural groups cannot be assumed to be a direct link. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. Constructivism, inquiry learning, and ethnomathematics allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. Ethnomathematics also shows the relationship between mathematics and cultural anthropology.

Individually, and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to themselves personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of Elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in students' understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, consider the indicator:

> Critique the statement "The sine law and the cosine law only apply to non-right triangles".

It is important for students to realize that, although cumbersome, the cosine and sine laws are just as valid for right triangles. This helps students to understand that these laws are merely an extension of the trigonometric ratios. It is important for students to see that what are distinct sections from different courses, or even the same course, often serve to extend what they already know rather than creating completely new knowledge.

Critiquing statements is an effective way to assess students individually, as a small group, or as a large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important for students to use representations that are meaningful to them.

(NCTM, 2000, p. 140)

It is important, in learning mathematics, that students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a more concrete starting point. Therefore, teachers must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving surface area: What is the surface area of your computer? Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures), or both.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and other symbolic models). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another ("How could you represent what you've done here using mathematical symbols?") or by having students compare their representations with others in the class.

In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models.

Making connections also involves looking for patterns. For example, in outcome P20.10:

> Demonstrate understanding of arithmetic and geometric (finite and infinite) sequences and series.
>
> [CN, PS, R, T]

students should be exploring series and learning to identify the type of sequences and series (arithmetic, geometric, or other) based upon the patterns that the students identify within the sequences and series. In addition, students should be analyzing the patterns that distinguish the different types of sequences and series in order to develop strategies for writing general terms for the sequences and series, as well as for solving different types of problems related to sequences and series.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help to consolidate new learnings with prior knowledge, explore possible solutions, and apply learning to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of drill will vary among different learners. In addition, when used as homework, drill and practice frequently causes frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Pre-calculus 20, consider outcome P20.6:

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.

(NCTM, 2000, p. 139)

Characteristics of Good Math Homework

- *It is accessible to [students] at many levels.*
- *It is interesting both to [students] and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to, and types of, math homework offered to [students] throughout the year.*

(Adapted from Raphel, 2000, p. 75)

Expand and demonstrate understanding of factoring polynomial expressions including those of the form:

- $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$
- $a(f(x))^2 - b(f(x)) + c, a \neq 0$
- $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

where *a*, *b*, and *c* are rational numbers.

[CN, ME, R]

Before starting to look at different types of factoring, the students can be given a homework task to sort, in any way that makes sense to them, a set of expressions (which happen to all be expressions that can be factored). This will engage the students in thinking and looking deeply at the expressions and for their defining characteristics, which is an important skill needed for the students to learn to decide how to factor an expression. The students would then bring their sorting of the expressions to class and have a partner or small group try to work out the sorting rule(s) used by each person. A random selection of the sorts could then be shared with the entire class to discuss, followed by the teachers' sorting of the expressions (by type(s) of factoring). The class would then engage in an inquiry into what sorting rule(s) the teacher used, thus getting the students familiar with what to look for when factoring before they have begun to learn about factoring. This could then be followed by the naming of the different sets (e.g., difference of squares or binomial squares). Future homework could include the expanding of different sets of factors to look for patterns and connections between the factors and the expanded forms.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Feedback can take many different forms. Instead of saying, "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about ...?"

(Stiff, 2001, p. 70)

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.

(NCTM, 2000, p. 72)

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, function notation is something which the teacher will have to show and name for the students; however, first, the students could explore the ideas important for working with function notation.

It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences .... A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.

(Haylock & Cockburn, 2003, p. 18)

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploring of patterns and relationships
- sharing ideas and problems
- considering different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

What might you hear or see in a Grade 11 classroom that would indicate to you that students were developing a deep understanding?

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.

(Mills & Donnelly, 2001, p. xviii)

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

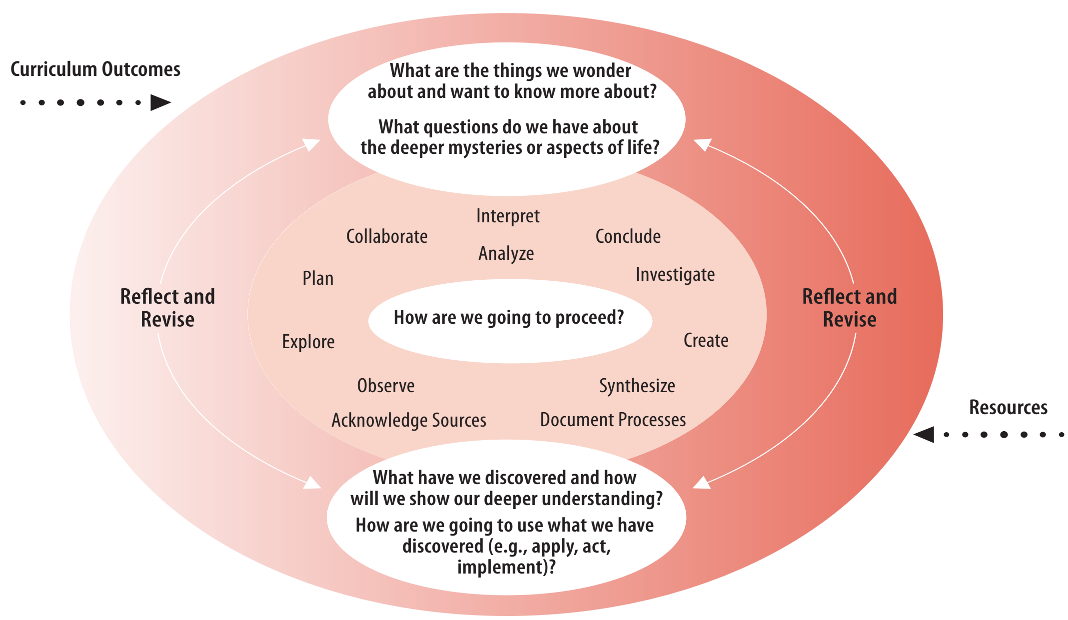
Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with parts of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic shows the cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, develop conclusions, document and reflect on learning, and generate new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students must understand this difference too.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry, and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When or why might you want to factor a polynomial?"
- "How do you know when you have an answer?"
- "Will this strategy work for all situations?"
- "How does your representation compare to that of your partner?"

Effective questions:

- *cause genuine and relevant inquiry into the important ideas and core content*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons*
- *spark meaningful connections with prior learning and personal experiences*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning. Questioning should also be used to encourage students to reflect on the inquiry process and on the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- help students make sense of the mathematics.
- are open-ended, whether in answer or approach; there may be multiple answers or multiple approaches.
- empower students to unravel their misconceptions.
- not only require the application of facts and procedures but encourage students to make connections and generalizations.
- are accessible to all students and offer an entry point for all students.
- lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.

(Schuster & Canavan Anderson, 2005, p.3)

Reflection and Documentation of Inquiry

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.

(Schuster & Canavan Anderson, 2005, p. 1)

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, models, works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Outcomes and Indicators

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.1 Demonstrate understanding of the absolute value of real numbers and equations and functions involving the absolute value of linear and quadratic functions. **[C, PS, R, T, V]**	a. Provide examples relevant to one's life, family, or community that illustrate different situations in which quantities referenced are positive, negative, or an absolute value and justify. b. Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a ($\|a\|$). c. Determine the absolute value of a real number. d. Order, with justification, a set of real numbers that includes the absolute value of one or more of the quantities. e. Explain, with the use of examples, how absolute value fits into the order of operations used on expressions involving real numbers. f. Determine the value of numerical expressions involving absolute value(s). g. Simplify expressions involving absolute value(s). h. Analyze, describe, and explain the relationship between the graph of $y = f(x)$ and $y = \|f(x)\|$. i. Create a table of values for $y = \|f(x)\|$ given $y = f(x)$. j. Sketch the graph of $y = \|f(x)\|$ given $y = f(x)$ and explain the reasoning. k. Develop and apply strategies for determining the intercepts, domain, and range of $y = \|f(x)\|$ given the equation of the function or its graph. l. Explain what the range of the function $y = \|f(x)\|$ reveals about the graph of the function. m. Develop, generalize, explain, and apply strategies for graphically determining (with and without the use of technology) the solution set of an equation involving absolute values of algebraic expressions. n. Develop, generalize, explain, and apply strategies for algebraically determining the solution set of an equation involving absolute values of algebraic expressions. o. Analyze and generalize conclusions about absolute value inequalities of the form $\|f(x)\| < 0$. p. Identify and correct errors in a solution to an absolute value equation.

Outcomes	Indicators
P20.1 continued	q. Solve situational questions involving absolute value functions or equations. r. Analyze and generalize the relationship between $\lvert x \rvert$ and $\sqrt{x^2}$ and between $\lvert f(x) \rvert$ and $\sqrt{(f(x))^2}$.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.2 Expand and demonstrate understanding of radicals with numerical and variable radicands including: • **computations** • **solving equations (limited to square roots and one or two radicals).** **[C, CN, ME, PS, R, T]**	a. Develop, generalize, explain, and apply strategies for expressing an entire radical (with numerical or variable radicand) as a mixed radical. b. Develop, generalize, explain, and apply strategies for expressing a mixed radical (with numerical or variable radicand) as an entire radical. c. Order a set of real numbers which includes radical expressions with numerical radicands. d. Develop, generalize, explain, and apply strategies for simplifying radical expressions (with numerical and/or variable radicands). e. Develop, generalize, explain, and apply strategies for rationalizing the denominator of rational expressions with monomial or binomial denominators. f. Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression. g. Verify and explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = \lvert x \rvert$, and $\sqrt{\boldsymbol{x}^2} \neq \pm \boldsymbol{x}$. h. Solve situational questions that involve radical expressions. i. Develop, explain, and apply strategies for determining the values of a variable for which a given radical expression is defined. j. Develop, explain, and apply strategies for determining non-permissible values (restrictions on values) for the variable in a radical equation. k. Develop, explain, and apply algebraic strategies for determining and verifying the roots of a radical equation. l. Explain why some roots determined in solving a radical equation are extraneous. m. Model and solve situational questions that involve radical equations.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.3 Expand and demonstrate understanding of rational expressions and equations (up to and including degree 2 numerators and denominators) including: • **equivalent forms of expressions** • **operations on expressions** • **solving equations that can be simplified to linear or quadratic equations.** **[C, CN, ME, R]**	a. Develop, verify, explain, and apply strategies for determining equivalent rational expressions. b. Compare the determining of equivalent rational expressions to determining equivalent rational numbers. c. Verify, with explanation, whether or not a given value is permissible for a given rational expression. d. Develop, explain, and apply strategies for determining the non-permissible values of a rational expression. e. Develop, explain, and apply strategies for simplifying rational expressions. f. Explain why the non-permissible values of a simplified rational expression must be stated as those of the original rational expression. g. Apply understanding of rational expressions to locate and correct errors in the simplification of a rational expression. h. Develop, verify, explain, and apply strategies for adding, subtracting, multiplying, and dividing rational expressions, including the determination of non-permissible values. i. Compare the performing of operations on rational expressions to performing the same operations on rational numbers. j. Develop, explain, and apply strategies for simplifying rational expressions that involve two or more operations on the rational expressions. k. Develop, explain, and apply algebraic strategies for determining the solution, including non-permissible values, of equations involving rational expressions. l. Explain why a value obtained in solving a rational equation may not be a solution of the equation. m. Model and solve situational questions involving rational expressions.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.4 Expand and demonstrate understanding of the primary trigonometric ratios including the use of reference angles ($0° \le \theta \le 360°$) and the determination of exact values for trigonometric ratios. **[C, ME, PS, R, T, V]**	a. Provide examples relevant to one's self, family, or community that illustrate the need to define a standard position for angles. b. Sketch an angle in standard position given the measure of the angle. c. Determine and justify, with or without sketching, the quadrant in which an angle in standard position terminates. d. Determine the reference angle for an angle in standard position. e. Analyze, describe, and generalize the relationship between the reference angles for angles (in standard positions) that are reflections of each other across both the x- and y- axes (e.g., 30° and 150°, or -60° and 60°). f. Sketch an angle in standard position given a point P(x, y) on the terminal arm of the angle. g. Develop, generalize, explain, and apply strategies for determining a point on the terminal arm of the angle in each quadrant that has the same reference angle as the angle with P(x, y) on its terminal arm. h. Develop, explain, and apply strategies for determining the distance between the origin and a point P(x, y) on the terminal arm of an angle. i. Develop, generalize, explain, and apply strategies for determining the value of sinθ, cosθ, and tanθ when given a point P(x, y) on the terminal arm of θ. j. Develop, generalize, explain, and apply strategies for determining sin θ, cos θ, and tan θ for quadrantal angles. k. Develop, generalize, explain, and apply strategies for determining the sign (without calculation or the use of technology) of sin θ, cos θ, or tan θ for a given value of θ. l. Develop, explain, and apply strategies for solving, for all values of θ, equations of the form $\sin \theta = a$ or $\cos \theta = a$, where $-1 \le a \le 1$, and equations of the form $\tan \theta = a$, where a is a real number. m. Analyze 30°- 60°- 90° and 45°- 45°- 90° triangles to generalize about the relationship between pairs of sides in such triangles in relation to the angles. n. Develop, generalize, explain, and apply strategies for determining the exact value of the sine, cosine, or tangent (without the use of technology) of an angle with a reference angle of 30°, 45°, or 60°.

Outcomes

P20.4 continued

Indicators

o. Describe and generalize the relationships and patterns in and among the values of the sine, cosine, and tangent ratios for angles from 0° to 360°.

p. Create and solve a situational question relevant to one's self, family, or community which involves a trigonometric ratio.

q. Identify angles for which the tangent ratio does not exist and explain why.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes

P20.5 Demonstrate understanding of the cosine law and sine law, including the ambiguous case.

[C, CN, PS, R, T]

Indicators

a. Provide a diagram or picture to illustrate a situation relevant to one's self, family, or community that involves a triangle without a right angle.

b. Develop, explain, and apply strategies for solving a non-right angle triangle using the primary trigonometric ratios.

c. Derive and explain a proof of the sine law or cosine law.

d. Provide an example of a situation relevant to one's self, family, or community that involves the need to consider the ambiguous case and provide a diagram or picture to illustrate the situation and explain why the ambiguous case needs to be considered.

e. Apply the sine law and/or cosine law to solve situational questions.

f. Critique the statement "For every possible pair of angles (whose sum is less than 180°) and line segment, a triangle can be constructed".

g. Critique the statement "The sine law and the cosine law only apply to non-right triangles".

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour	
Outcomes	**Indicators**
P20.6 Expand and demonstrate understanding of factoring polynomial expressions including those of the form: • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ • $a(f(x))^2 - b(f(x)) + c, a \neq 0$ • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ **where *a*, *b*, and *c* are rational numbers.** **[CN, ME, R]**	a. Develop, generalize, explain, and apply strategies for factoring polynomial expressions of the form: • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$, *a* and *b* are real numbers • $ca^2x^2 - cb^2y^2, a \neq 0, b \neq 0$, *a*, *b*, and *c* are real numbers • $a(f(x))^2 - b(f(x)) + c, a \neq 0$, *a*, *b*, and *c* are real numbers • $da(f(x))^2 - db(f(x)) + dc, a \neq 0$, *a*, *b*, *c*, and *d* are real numbers • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$, *a* and *b* are real numbers • $da^2(f(x))^2 - db^2(g(y))^2, a \neq 0, b \neq 0$, *a*, *b*, and *d* are real numbers b. Verify, with explanation, whether or not a given binomial is a factor for a given polynomial.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour	
Outcomes	**Indicators**
P20.7 Demonstrate understanding of quadratic functions of the form **$y = ax^2 + bx + c$ and of their graphs, including:** • **vertex** • **domain and range** • **direction of opening** • **axis of symmetry** • **x- and y-intercepts.** **[CN, PS, R, T, V]**	a. Generalize a rule from sets of graphs, using inductive reasoning, and explain about how different values of *a* (including 1, 0, and -1) transform the graph of $y = ax^2$. b. Generalize a rule from sets of graphs, using inductive reasoning, and explain about how different values of *q* (including 0) transform the graph of $y = x^2 + q$. c. Generalize a rule from sets of graphs, using inductive reasoning, and explain how different values of *p* (including 0) transform the graph of $y = (x - p)^2$. d. Develop, generalize, explain, and apply strategies for determining the coordinates of the vertex, the domain and range, the axis of symmetry, x- and y- intercepts, and direction of opening of the graph of the function $f(x) = a(x-p)^2 + q$ without the use of technology. e. Develop, explain, and apply strategies for graphing functions of the form $f(x) = a(x - p)^2 + q$ by applying transformations related to the values of *a*, *p*, and *q*. f. Develop, explain, and apply strategies (that do not require graphing or the use of technology) for determining whether a quadratic function will have zero, one, or two x-intercepts. g. Develop, explain, and apply strategies for writing a quadratic function in the form of $y = a(x - p)^2 + q$ that represents a given graph or set of characteristics of a graph.

Outcomes	Indicators
P20.7 continued	h. Develop, generalize, explain, verify, and apply a strategy (including completing the square) for writing a quadratic function in the form $y = ax^2 + bx + c$ in the form $y = a(x - p)^2 + q$. i. Using knowledge about completing the square, identify and correct errors in a given example of completing the square. j. Develop, generalize, explain, and apply strategies for determining the coordinates of the vertex, the domain and range, the axis of symmetry, x- and y- intercepts, and direction of opening of the graph of a function in the form $y = ax^2 + bx + c$. k. Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$. l. Write a quadratic function that models a given situation and explain any assumptions made. m. Analyze quadratic functions (with or without the use of technology) to answer situational questions.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.8 Demonstrate understanding of quadratic equations including the solution of: • **single variable equations** • **systems of linear-quadratic and quadratic-quadratic equations in two variables.** **[C, CN, PS, R, T, V]**	*Note: It is intended that the quadratic equations be limited to those that correspond to quadratic functions.* a. Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the x-intercepts of the graph of the quadratic function. b. Derive the quadratic formula, using deductive reasoning. c. Apply strategies for solving quadratic equations of the form $ax^2 + bx + c = 0$ including: • determining square roots • factoring • completing the square • applying the quadratic formula • graphing its corresponding function, with and without the use of technology. d. Explain different strategies for verifying the solution to a quadratic equation.

Outcomes	Indicators
P20.8 continued	e. Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one, or no real roots; and relate this knowledge to the number of zeros that the corresponding quadratic function will have. f. Apply knowledge of quadratic equations and functions to identify and correct any errors within a solution to a quadratic equation. g. Solve situational questions involving the writing and solving of quadratic equations. h. Match systems of linear-quadratic and quadratic-quadratic functions to situations. i. Develop, generalize, explain, and apply strategies for solving systems of linear-quadratic and quadratic-quadratic functions, including: • graphically • algebraically • with the use of technology. j. Explain the meaning of the intersection point of a system of linear-quadratic or quadratic-quadratic equations in terms of the situation being modeled. k. Illustrate and explain how a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions. l. Solve situational questions by using systems of linear-quadratic or quadratic-quadratic equations.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.9 Expand and demonstrate understanding of inequalities including: • **one-variable quadratic inequalities** • **two-variable linear and quadratic inequalities.** **[C, CN, PS, T, V]**	a. Develop, generalize, explain, and apply strategies for determining the solution region for two-variable linear or two-variable quadratic inequalities. b. Explain, using examples, how test points can be used to determine the solution region that satisfies a two-variable inequality. c. Explain, using examples, when a solid or broken line should be used in the graphic solution of a two-variable inequality. d. Explain what the solution region for a two-variable inequality means. e. Solve a situational question that involves a two-variable inequality.

Outcomes	Indicators
P20.9 continued	f. Develop, generalize, explain, and apply strategies, such as case analysis, graphing, roots and test points, or sign analysis, to solve one-variable quadratic inequalities. g. Model and solve a situational question that involves a one-variable quadratic inequality. h. Interpret the solution to a situational question that involves a one-variable quadratic inequality.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.10 Demonstrate understanding of arithmetic and geometric (finite and infinite) sequences and series. **[CN, PS, R, T]**	a. Identify assumptions made in determining that a sequence or series is either arithmetic or geometric. b. Provide an example of a sequence that follows an identifiable pattern, but that is neither arithmetic nor geometric. c. Provide an example of an arithmetic or geometric sequence that is relevant to one's self, family, or community. d. Generate arithmetic or geometric sequences from provided information. e. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, d, n, or t_n in situational questions that involve arithmetic sequences. f. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, d, n, or S_n in situational questions that involve arithmetic series. g. Solve situational questions that involve arithmetic sequences and series. h. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, r, n, or t_n in situational questions that involve geometric sequences. i. Develop, generalize, explain, and apply a rule and other strategies for determining the values of t_1, a, r, n, or S_n in situational questions that involve geometric series. j. Develop, generalize, and explain a rule and strategies for determining the sum of an infinite geometric series and apply this knowledge to the solving of situational questions. k. Analyze a geometric series to determine if it is convergent or divergent and explain the reasoning.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P20.11 Demonstrate understanding of reciprocal functions of: • **linear functions** • **quadratic functions.** **[CN, R, T, V]**	a. Describe the relationship between a function and its reciprocal. b. Apply knowledge of rational expressions to determine non-permissible values for reciprocal functions. c. Analyze and describe the relationship between vertical asymptotes and non-permissible values. d. Develop, explain, and apply strategies for graphing (with or without the use of technology) $y = \frac{1}{f(x)}$ given either the graph or equation for $y = f(x)$ where $f(x)$ is a polynomial of degree ≤ 2. e. Develop, explain, and apply strategies for graphing (with or without the use of technology) $y = f(x)$ given either the graph or equation for $y = \frac{1}{f(x)}$ where $f(x)$ is a polynomial of degree ≤2. f. Sketch the graph of a function in the form $y = \frac{1}{f(x)}$. g. Analyze reciprocal functions to describe the end behaviour of the functions.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assembling evidence from a variety of sources is more likely to yield an accurate picture.

(NCTM, 2000, p. 24)

Assessment involves the systematic collection of information about student learning with respect to:

- Achievement of provincial curriculum outcomes
- Effectiveness of teaching strategies employed
- Student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Pre-calculus 20. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment should not merely be done to students; rather it should be done for students.

(NCTM, 2000, p. 22)

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curricular outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

What are examples of assessments as learning that could be used in Pre-calculus 20 and what would be the purpose of those assessments?

Assessment should become a routine part of the ongoing classroom activity rather than an interruption.

(NCTM, 2000, p. 23)

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The various types of assessments should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the content of Pre-calculus 20.

Glossary

Case Analysis: A method for solving inequalities in which the possible combinations of signs of the individual factors are considered separately as individual cases and then the results are combined logically. For example, if $ab<0$ then the two possible cases of $a<0$ and $b>0$, or $a>0$ and $b<0$ are solved separately, and then the resulting solutions are combined so that both are satisfied by the final solution.

Convergent Series: A series is convergent if, as the number of terms used to determine the series value increases, the sum gets closer and closer to a particular value.

Deductive Reasoning: Reasoning that moves from a general known (has been proven or assumed) to a specific conclusion. For example, if you know that the sum of the measures of the angles in a triangle is always 180°, then you can deductively determine the measure of the third angle in a triangle when you know the measures of the other two angles.

Direct Measurement: Measurement obtained by physically using a measurement tool, such as a metre stick, thermometer, or scale, on the object or quantity being measured.

Divergent: A series is divergent if, as the number of terms used to determine the series value increases, the sum oscilates between values or grows without bound (positively or negatively).

Equation: An equation is a statement which says that one expression or quantity is equal to another. It is an algebraic statement containing two expressions linked by an equals sign.

Expression: A finite combination of symbols (numeric and/or variable) related through mathematical operations, excluding equalities and inequalities.

Function: A special type of relation which exists between each number in one set and just one number in a second set. The first set is referred to as the domain of the function, while the second set is called the range of the function.

Generalize: The process of describing, in general, patterns and/or processes from specific examples and cases. Frequently, generalizing is an inductive process but it can also involve deductive proof of the pattern or process.

Graphic Organizer: A pictorial or other concrete representation of knowledge, concepts, and/or ideas and connections among them.

Indirect Measurement: The use of proportions and proportional relationships, such as similar triangles, the Pythagorean Theorem, and the trigonometric ratios, to determine a measurement that cannot or has not been measured directly. It is the determining of a measurement based on other known measurements rather than by physically measuring.

Inductive Reasoning: Reasoning that infers a general conclusion or rule from specific cases. For example, if a number of triangles are examined and it is found that the sum of the interior angles for each triangle is 180°, then one might inductively conclude (without proof) that the sum of the measures of the interior angles of any triangle is 180°.

Prove: Demonstration that a mathematical conclusion is true through a logical argument with justification for each step. The demonstration is such that it accounts for all possible scenarios.

Quadrantal Angle: An angle in standard position with its terminal side on a coordinate axis.

Referent: A personally determined concrete or physical approximation of a quantity or unit of measurement. For example, some people use the width of their thumb as a referent for one inch.

Relation: A description of how two sets of things can be connected to each other.

Sign Analysis: An analysis of the sign of an expression or function over a given interval by composing the signs of the individual factors of the expression or function.

Situational Quesitons: Mathematical questions that are asked within the context of a particular situation. Situational questions may be actual problems (something which the student does not yet know how to solve) or practice (something which the student has seen examples of how to solve).

Test Point: Point used to determine the sign of an expression or function over the interval in which the point is a member.

Transformation: Replacement of the variables in an algebraic expression by their values in terms of another set of variables. The resulting change in the graph of a function when one or more numeric coefficients in the equation of the function are changed and vice versa.

Verify: Demonstration that a particular result or set of results satisfies an equation. Verification can also be showing that, for a particular case(s), a generalization works.

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Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Pre-calculus 20

1. Please indicate your role in the learning community

☐ parent ☐ teacher ☐ resource teacher

☐ guidance counsellor ☐ school administrator ☐ school board trustee

☐ teacher librarian ☐ school community council member

☐ other ____

What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

☐ print

☐ online

b) Please indicate which format(s) of the curriculum you prefer:

☐ print

☐ online

3. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
appropriate for its intended purpose	1	2	3	4
suitable for your use	1	2	3	4
clear and well organized	1	2	3	4
visually appealing	1	2	3	4
informative	1	2	3	4

4. Explain which aspects you found to be:

Most useful:

Least useful:

5. Additional comments:

6. Optional:

   Name: ____

   School: ____

   Phone: __________________ Fax: __________________

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Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

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